I. Factoring.
\[ x^2 - a^2 = (x - a)(x + a), \quad (x + a)^2 = x^2 + 2xa + a^2 \]
\[ (x - a)^2 = x^2 - 2xa + a^2, \quad x^3 - a^3 = (x - a)(x^2 + xa + a^2) \]
\[ x^3 + a^3 = (x + a)(x^2 - xa + a^2) \]

FOIL!!! \((x + a)(x + b) = x^2 + 2ab + b^2\)

II. Exponents
\[ a^0 = 1 \quad a^a \cdot a^b = a^{a+b} \]
\[ (a^x)^y = a^{xy} \quad a^{-x} = \frac{1}{a^x} \]
\[ (ab)^x = a^{x}b^{x} \quad \left(\frac{a}{b}\right)^x = \frac{a^{x}}{b^{x}} \]

III. Radicals And Fractional Exponents
If \(n\) is odd and then \(\sqrt[n]{a} = b\) if \(b^n = a\)
If \(n\) is odd and then \(\sqrt[n]{a^x} = a^{\frac{x}{n}}\)
In both cases \(\sqrt[n]{a}\) is called the principal nth-root of \(a\)
If \(n\) is odd and then \(\sqrt[n]{a} = a\)
If \(n\) is even and then \(\sqrt[n]{a^x} = |a| \quad \sqrt[n]{a} = \sqrt[n]{|a|}\)
\[ a^\frac{1}{x} \text{ means } \sqrt[n]{a} \quad a^\frac{m}{x} \text{ means } \sqrt[n]{a^m} \quad (\sqrt[n]{a})^m \]

IV. Quadratic Formula, Distance, midpoint and Circles.
If \(ax^2 + bx + c = 0\), with \(a \neq 0\) then \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

Given two points, \((x_0, y_0)\) and \((x_1, y_1)\) the distance between the points is \(\sqrt{(x_1-x_0)^2 + (y_1-y_0)^2}\),
the midpoint is \(\left(\frac{x_1+x_0}{2}, \frac{y_1+y_0}{2}\right)\), (average the coordinates).

The equation of a circle with center \((h, k)\) and radius \(r > 0\) is \((x - h)^2 + (y - k)^2 = r^2\).

V. Lines and slope.
not parallel to y axis
Given two points, \((x_0, y_0)\) and \((x_1, y_1)\) with \(x_0 \neq x_1\) the slope of the line between the two points is
\[ m = \frac{y_1 - y_0}{x_1 - x_0} \quad \text{rise} \quad \text{run} = m \]

and the point-slope equation of the line through \((x_0, y_0)\) and \((x_1, y_1)\) is \(y - y_0 = m(x - x_0)\).

The slope-intercept equation of the line through \((x_0, y_0)\) and \((x_1, y_1)\) is \(y = mx + b\) with \((0, b)\) the y intercept.

Two lines are parallel if they have the same slope \(m_1 = m_2\) and two lines are perpendicular if one slope is the negative reciprocal of the other \(m_1 = -\frac{1}{m_2}\)

parallel to y axis
If \(x_0 = x_1\), (line determined by \((x_0, y_0)\) and \((x_1, y_1)\) is parallel to y axis) and the slope is undefined. The equation of the line through \((x_0, y_0)\) and \((x_1, y_1)\) is \(x = x_0\).

VI. The Domain, Range And Function Composition
The domain of a function is where the function is defined.
If the domain is not specified it is taken to be those real numbers for which the function makes sense (e.g. domains not zero and inside sqrt’s not negative). The range is the set of image points i.e. the set of all \(f(x)\) where \(x\) is in the domain.
Also if the graph \(G\) is known then the domain of \(f\) is the projection of \(G\) on the X axis and the range of \(f\) is the projection of \(G\) on the Y axis

(fog)(x) means \(f(g(x))\) and \((gof)(x)\) means \(g(f(x))\)

VII. Point Symmetry And Set Symmetry
Given a point \((x, y)\), the reflection of this point about the y-axis is \((-x, y)\), the reflection of this point about the origin is \((-x, -y)\), and the reflection of this point about the x-axis is \((x, -y)\). A set of points \(G\) (usually the graph of an equation) is called symmetric about the y-axis if \(G\) is the same as its reflection about the y-axis. A set of points \(G\) (usually the graph of an equation) is called symmetric about the origin if \(G\) is the same as its reflection about the origin. A set of points \(G\) is called symmetric about the x-axis. If \(G\) is the same as its reflection about the x-axis.

VIII. Functions And Symmetry
If the graph \(G\) is that of a function \(f(x)\) the graph \(G\) will be symmetric about the y-axis if and only if \(f(x) = f(-x)\), i.e. an even function. If the graph \(G\) is that of a function \(f(x)\) the graph \(G\) will be symmetric about the origin if and only if \(-f(-x) = f(-x)\), or equivalently \(f(x) = -f(-x)\) and is called an odd function. In each case simplify \(-f(-x)\) and see if it equals either \(-f(x)\) (odd) or \(f(x)\) (even).

IX. Increasing And Decreasing functions
\(f(x)\) is increasing on \((a, b)\) (NOT \([a, b]\)) if whenever \(a < t < u < b\) then \(f(t) < f(u)\) i.e. the graph of if slants “up on \((a, b)\)” \(f(x)\) be is decreasing on \((a, b)\) if whenever \(a < t < u < b\) then \(f(t) > f(u)\) i.e. the graph of if slants “down on \((a, b)\)”

X. Shifting
If the graph of a function is \(G\) we geometrically shift \(G\) in natural ways and change the function so that the modified function gives the shift.
up a units : \(f(x) + a\)
down a units : \(f(x) - a\)
left a units : evaluate the function at \(x + a\) ; (replace \(x \) with \(x + a\))
right a units : evaluate the function at \(x - a\) (replace \(x \) with \(x - a\))
Reflect about \(y - axis\) ; evaluate the function at \(-x\) ; i.e. replace \(x\) with \(-x\)
Reflect about the \(x - axis\) ; negate the function i.e. replace the function with \(-f(x)\)

XI. Average Value(AV) (Or Difference Quotient)(DQ)
of A Function
Let \(f(x)\) be defined on \([a, b]\) The AV of \(f(x)\) on \([a, b]\) is
\[ \frac{b - a}{h} \]
Let \(f(x)\) be defined on \([x, x + h]\) The DQ of \(f(x)\) on \([x, x + h]\) is
\[ \frac{f(x + h) - f(x)}{h} \]
XII. Inverse Functions

If \( f(x) \) is a one to one function then it has an inverse \( f^{-1}(x) \).

To find the inverse (4 step process) switch \( x,y \) in \( y = f(x) \), solve for \( y \) in terms of \( x \) and that expression for \( y \) is \( f^{-1}(x) \).

Also the domain of \( f(x) \) is the range of \( f^{-1}(x) \) and the domain of \( f^{-1}(x) \) is the range of \( f(x) \).

The graph of \( f^{-1}(x) \) may be obtained rotating the graph of \( f \) about the line \( y = x \) (180 degree perpendicular rotation).

XIII. Quadratics And Parabolas

If \( y = f(x) = ax^2+bc+c \) with \( a \neq 0 \). The graph is a parabola which opens up if \( a > 0 \) and opens down if \( a < 0 \).

The Vertex is \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \) and the Axis of Symmetry is \( x = \frac{-b}{2a} \).

The standard form: \( y = a(x-h)^2 + k \) has \( (h,k) \) as the vertex, and \( x = h \) is the Axis of Symmetry. Solve a max/min question by finding the vertex.

XIV. Finding The Roots (or Zeros) of a Polynomial

If \( P(x) = a_nx^n + \ldots + a_0 \) is a polynomial with integer coefficients then any rational root of \( P(x) \) must have the form \( \pm \frac{a}{b} \) where \( a \) divides \( a_0 \) and \( b \) divides \( a_n \).

If \( P(x) \) is a Polynomial Then \( a \) is a root of \( P(x) \) (i.e. \( P(a) = 0 \)) if and only if \( x - a \) divides \( P(x) \). We can use synthetic or long division to factor \( x - a \) out of \( P(x) \).

Use the above two techniques to completely factor \( P(x) \).

XV. Vertical(VA), Horizontal(HA), And Slant (SA) Asymptotes.

Let \( R(x) = \frac{P(x)}{Q(x)} \) be rational (Must Be Put In Lowest terms!). Then

1. \( x = \) any root (zero) of \( Q(x) \) is a VA
2. If \( \deg P(x) < \deg Q(x) \) then \( y = 0 \) is a HA
3. If \( \deg P(x) = \deg Q(x) \) then \( y = \) leading coefficient of \( P \) is an HA
4. A SA will exist only if \( \deg P(x) = \deg Q(x) + 1 \) and then in this case divide \( P \) by \( Q \) (ignore remainder) to obtain a line which is the SA

XVI. Solving Polynomial and Rational Inequalities

Solving Polynomial Inequalities

Let \( P(x) \) and \( Q(x) \) be polynomials. To solve

\[ T(x) = \frac{P(x)}{Q(x)} \quad (\leq, <, >, \text{ or } >) \quad 0 \]

1. Find all the zeros of \( P(x) \) and \( Q(x) \) and plot them on the real line
2. The roots naturally divide the line into parts and on the inside of each part \( T(x) \) is always positive or negative
3. Choose a point \( x \) on the INSIDE of each part and compute \( T(x) \) (which is + or -) The Parts that are positive form a solution to \( T(x) > 0 \) and the Parts that are negative form a solution to \( T(x) < 0 \). The solutions to \( T(x) = 0 \) include the endpoints where \( P(x) = 0 \) but NOT the places where \( Q(x) = 0 \).

XVI. End behavior, crossing.touching of zero’s

If \( f(x) \) is a polynomial function, then we note the degree \( n \) and the lead coefficient \( a \).

If \( a > 0 \) then the end behavior is UP/UP if \( n \) is even, while it is DOWN/UP if \( n \) is odd.

If \( a < 0 \) then the end behavior is DOWN/DOWN if \( n \) is even, while it is UP/DOWN if \( n \) is odd.

A zero/root \( c \) of \( f(x) \) has multiplicity \( k \) if \( (x-c)^k \) is the largest power of \( x-c \) that is a factor of \( f(x) \). The graph of \( f(x) \) crosses the \( x \)-axis at all zero’s of odd multiplicity and it touches the \( x \)-axis at all zero’s of even multiplicity.

The graph has at most \( n \) zero’s and at most \( n-1 \) turning points.

XVII. Direct , Inverse And Mixed Proportionality

If \( y \) is directly proportional to \( x \) if \( y = kx \) for some \( k \neq 0 \)

If \( y \) is inversely proportional to \( x \) if \( y = \frac{k}{x} \) for some \( k \neq 0 \)

In a problem we might be given one sample pair for \( x,y \) so we can sub them in to solve for \( k \). We sometimes call all this varying rather than proportional.

XVIII. Logs

\[ \log_{a} b = c \text{ means } b^c = a \text{ where } b \text{ is the base}. \]

The first is called the log form, the second the exponential form. If \( x \) is in the exponent, solve the question by converting to log form. If \( x \) is inside the log, solve by combining into a single log and then converting to exponential form.

\[ \log(z) \] is only defined if \( z > 0 \) (hence in a “what is the domain” question, inside a log has to be greater than 0). The domain of the exponential function is \((-\inf, \inf)\).

\[ \log(xy) = \log(x) + \log(y) \] (any base)

\[ \log(x^p) = p \log(x) \] (any base)

\[ \log(z^\frac{1}{p}) = \log(z) - \log(y) \] (any base)

\[ \log_{b}(a) = \log_{b}(c) - \log_{b}(z) \] (very useful for solving exponential equations)

\[ \ln x \] means \( \log_{e} x \)

\[ \ln e^x = x \] (very useful for solving logarithmic equations)

\[ \ln 1 \] and \( \ln e \) are in your calculators. for other bases \( b \) use

\[ \log_{b}(a) = \frac{\ln a}{\ln b} = \frac{\log_{a}(c)}{\log_{b}(c)} \] (Change of base)

XIX. Money And Interest Rates

Period or Complex Compounding

\( A = \) amount at end of investment period, \( P= \) principal, \( r = \) yearly simple interest rate, \( t = \) time invested and \( n = \) number of investment periods. Then

\[ A = P(1 + \frac{r}{n})^{nt} \]

Continuous Compounding

\[ A = Pe^{rt} \]

XX. Exponential growth/decay

If \( A(t) \) is a quantity changing with time and it grows or decays exponentially means that it has an equation like \( A(t) = A_0 e^{kt} \).

This is a bit like direct/inverse proportion. If we are told that the quantity varies exponentially and what it is at time \( t = 0 \) (this is \( A_0 \) and what the growth rate is (i.e. \( k \), then we can just compute \( A(t) \) at other times \( t \).

If we are told the value of \( A_0 \) and what the value of \( A(t) \) is at some other value of \( t \), then we can sub in those values and solve for \( k \) (using logs). Then we can say what the value of \( A(t) \) will be at other values of \( t \).