1. The addition below is incorrect. What is the largest digit that can be changed to make the addition correct?

$$\begin{array}{r} 6\ 4\ 1 \\ 8\ 5\ 2 \\ +\ 9\ 7\ 3 \\ \hline 2\ 4\ 5\ 6 \end{array}$$

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2. Each day Walter gets \$3 for doing his chores or \$5 for doing them exceptionally well. After 10 days of doing his chores daily, Walter has received a total of \$36. On how many days did Walter do them exceptionally well?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

3. $\dfrac{(3!)!}{3!} =$

**(A)** 1 **(B)** 2 **(C)** 6 **(D)** 40 **(E)** 120

4. Six numbers from a list of nine integers are 7, 8, 3, 5, 9, and 5. The largest possible value of the median of all nine numbers in this list is

**(A)** 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

5. Given that $0 < a < b < c < d$, which of the following is the largest?

**(A)** $\dfrac{a+b}{c+d}$ **(B)** $\dfrac{a+d}{b+c}$ **(C)** $\dfrac{b+c}{a+d}$ **(D)** $\dfrac{b+d}{a+c}$ **(E)** $\dfrac{c+d}{a+b}$

6. If $f(x) = x^{(x+1)}(x+2)^{(x+3)}$ then $f(0) + f(-1) + f(-2) + f(-3) =$

**(A)** $-8/9$ **(B)** 0 **(C)** $8/9$ **(D)** 1 **(E)** $10/9$

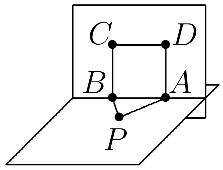
7. A father takes his twins and a younger child out to dinner on the twins' birthday. The restaurant charges \$4.95 for the father and \$0.45 for each year of a child's age, where age is defined as the age at the most recent birthday. If the bill is \$9.45, which of the following could be the age of the youngest child?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, then $r =$

(A) $-\log_2 5$ (B) $\log_5 2$ (C) $\log_{10} 5$ (D) $\log_2 5$ (E) $\frac{5}{2}$

9. Triangle $PAB$ and square $ABCD$ are in perpendicular planes. Given that $PA = 3$, $PB = 4$, and $AB = 5$, what is $PD$?




(A) 5 (B) $\sqrt{34}$ (C) $\sqrt{41}$ (D) $2\sqrt{13}$ (E) 8

10. How many line segments have both their endpoints located at the vertices of a given cube?

(A) 12 (B) 15 (C) 24 (D) 28 (E) 56

11. Given a circle of radius 2, there are many line segments of length 2 that are tangent to the circle at their midpoints. Find the area of the region consisting of all such line segments.

(A) $\pi/4$ (B) $4 - \pi$ (C) $\pi/2$ (D) $\pi$ (E) $2\pi$

12. A function $f$ from the integers to the integers is defined as follows:

$$f(n) = \begin{cases} n+3 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Suppose $k$ is odd and $f(f(f(k))) = 27$. What is the sum of the digits of $k$?

(A) 3 (B) 6 (C) 9 (D) 12 (E) 15

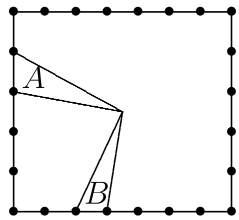
13. Sunny runs at a steady rate, and Moonbeam runs $m$ times as fast, where $m$ is a number greater than 1. If Moonbeam gives Sunny a head start of $h$ meters, how many meters must Moonbeam run to overtake Sunny?

(A) $hm$ (B) $\frac{h}{h+m}$ (C) $\frac{h}{m-1}$ (D) $\frac{hm}{m-1}$ (E) $\frac{h+m}{m-1}$

14. Let $E(n)$ denote the sum of the even digits of $n$. For example, $E(5681) = 6 + 8 = 14$. Find $E(1) + E(2) + E(3) + \cdots + E(100)$.

(A) 200 (B) 360 (C) 400 (D) 900 (E) 2250

15. Two opposite sides of a rectangle are each divided into $n$ congruent segments, and the endpoints of one segment are joined to the center to form triangle $A$. The other sides are each divided into $m$ congruent segments, and the endpoints of one of these segments are joined to the center to form triangle $B$. [See figure for $n = 5$, $m = 7$.] What is the ratio of the area of triangle $A$ to the area of triangle $B$?

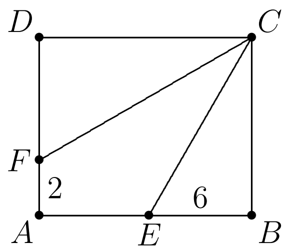



**(A)** 1 **(B)** $m/n$ **(C)** $n/m$ **(D)** $2m/n$ **(E)** $2n/m$

16. A fair standard six-sided die is tossed three times. Given that the sum of the first two tosses equals the third, what is the probability that at least one "2" is tossed?

**(A)** $\frac{1}{6}$ **(B)** $\frac{91}{216}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{8}{15}$ **(E)** $\frac{7}{12}$

17. In rectangle $ABCD$, angle $C$ is trisected by $\overline{CF}$ and $\overline{CE}$, where $E$ is on $\overline{AB}$, $F$ is on $\overline{AD}$, $BE = 6$, and $AF = 2$. Which of the following is closest to the area of the rectangle $ABCD$?

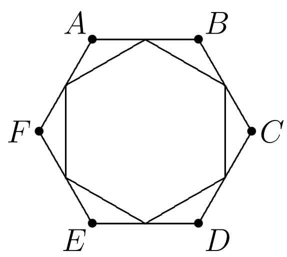



**(A)** 110 **(B)** 120 **(C)** 130 **(D)** 140 **(E)** 150

18. A circle of radius 2 has center at $(2, 0)$. A circle of radius 1 has center at $(5, 0)$. A line is tangent to the two circles at points in the first quadrant. Which of the following is closest to the $y$-intercept of the line?

**(A)** $\sqrt{2}/4$ **(B)** $8/3$ **(C)** $1+\sqrt{3}$ **(D)** $2\sqrt{2}$ **(E)** 3

19. The midpoints of the sides of a regular hexagon $ABCDEF$ are joined to form a smaller hexagon. What fraction of the area of $ABCDEF$ is enclosed by the smaller hexagon?



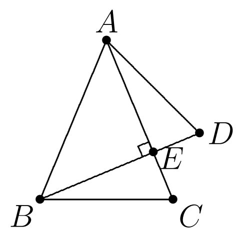

**(A)** $\frac{1}{2}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{\sqrt{3}}{2}$

20. In the $xy$-plane, what is the length of the shortest path from $(0, 0)$ to $(12, 16)$ that does not go inside the circle $(x-6)^2 + (y-8)^2 = 25$?

**(A)** $10\sqrt{3}$ **(B)** $10\sqrt{5}$ **(C)** $10\sqrt{3} + \frac{5\pi}{3}$ **(D)** $40\frac{\sqrt{3}}{3}$ **(E)** $10 + 5\pi$

21. Triangles $ABC$ and $ABD$ are isosceles with $AB = AC = BD$, and $\overline{BD}$ intersects $\overline{AC}$ at $E$. If $\overline{BD} \perp \overline{AC}$, then $\angle C + \angle D$ is

**(A)** $115^\circ$ **(B)** $120^\circ$ **(C)** $130^\circ$ **(D)** $135^\circ$
**(E)** not uniquely determined




22. Four distinct points, $A$, $B$, $C$, and $D$, are to be selected from 1996 points evenly spaced around a circle. All quadruples are equally likely to be chosen. What is the probability that the chord $\overline{AB}$ intersects the chord $\overline{CD}$?

**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

23. The sum of the lengths of the twelve edges of a rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

**(A)** 776 **(B)** 784 **(C)** 798 **(D)** 800 **(E)** 812

24. The sequence

$$1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, \ldots$$

consists of 1's separated by blocks of 2's with $n$ 2's in the $n^{\text{th}}$ block. The sum of the first 1234 terms of this sequence is

**(A)** 1996 **(B)** 2419 **(C)** 2429 **(D)** 2439 **(E)** 2449

25. Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have?

**(A)** 72 **(B)** 73 **(C)** 74 **(D)** 75 **(E)** 76

26. An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:

   (a) the selection of four red marbles;

   (b) the selection of one white and three red marbles;

   (c) the selection of one white, one blue, and two red marbles; and

   (d) the selection of one marble of each color.

   What is the smallest number of marbles satisfying the given condition?

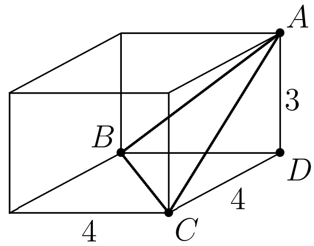
   **(A)** 19 **(B)** 21 **(C)** 46 **(D)** 69 **(E)** more than 69

27. Consider two solid spherical balls, one centered at $(0, 0, \frac{21}{2})$ with radius 6, and the other centered at $(0, 0, 1)$ with radius $\frac{9}{2}$. How many points $(x, y, z)$ with only integer coordinates (lattice points) are there in the intersection of the balls?

   **(A)** 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

28. On a $4 \times 4 \times 3$ rectangular parallelepiped, vertices $A$, $B$, and $C$ are adjacent to vertex $D$. The perpendicular distance from $D$ to the plane containing $A$, $B$, and $C$ is closest to

   **(A)** 1.6 **(B)** 1.9 **(C)** 2.1 **(D)** 2.7 **(E)** 2.9




29. If $n$ is a positive integer such that $2n$ has 28 positive divisors and $3n$ has 30 positive divisors, then how many positive divisors does $6n$ have?

   **(A)** 32 **(B)** 34 **(C)** 35 **(D)** 36 **(E)** 38

30. A hexagon inscribed in a circle has three consecutive sides each of length 3 and three consecutive sides each of length 5. The chord of the circle that divides the hexagon into two trapezoids, one with three sides each of length 3 and the other with three sides each of length 5, has length equal to $m/n$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

   **(A)** 309 **(B)** 349 **(C)** 369 **(D)** 389 **(E)** 409