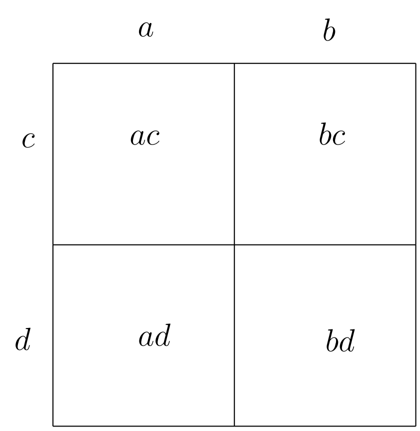
Hello. First I want to thank Tim Corica for asking me to write about something I love, namely lighting mathematical fires in middle and elementary school children. During the last two months and the next three months I have the delightful task of working two hours each week with 20 high-flying fourth and fifth graders at Barringer Academic Center in Charlotte, North Carolina in partnership with the Advanced Studies Department. The topics are typically from elementary and middle school: fractions, integer arithmetic, area, puzzles, spacial geometry. What the problems have in common is that they all have an arithmetic entry point, and they all include something mysterious. I want students to quiz one another with questions like 'How did he do that?'

The Area Model for multiplication represents a lovely way to view the distribution property of real numbers. This property is the link between addition and multiplication.

What is the area model for multiplication? It is a visual model that represents the product of two sums of numbers as the area of a suitably chosen rectangle. Below are several examples.

The distributive law. Consider the problem of computing the product $(a + b)(c + d)$.



What this shows is that an $a + b$ by $c + d$ rectangle can be partitioned into four rectangular regions with areas ac, bc, ad and bd, thus proving that

$(a+b)(c+d) = ab + bc + ad + bd$.

I asked my students to compute the product 216×23 in vertical format.

$$\begin{array}{r} 216 \\ \underline{\times 23} \\ 648 \\ \underline{4320} \\ 4968 \end{array}$$

Then I asked them the area of a 23×216 rectangle. Finally I challenged them to find the area of the six rectangles in into which we can divide the large rectangle.

	200	10	6	
3	600	30	18	648
20	4000	200	120	4320

Using negative numbers. Next, I asked my students to build a rectangle that measures 216×19 and compute its area.

	200	10	6
20	4000	200	120
-1	-200	-10	-6

Then I could show them that the area model works beautifully with negative numbers. That is because it is really just another way to look at the distribution of multiplication over addition. Thus $216 \times 19 = (200+10+6)(20-1) = 4000 + 200 + 120 - 200 - 10 - 6 = 4104$.

Multiplying polynomials. The area model works well with polynomials also. For example, compute $(2x^2 + x + 6)(2x + 3)$.

	$2x^2$	x	6
3	$6x^2$	$3x$	18
$2x$	$4x^3$	$2x^2$	$12x$

So, $(2x^2+x+6)(2x+3) = 4x^3+2x^2+12x+6x^2+3x+18 = 4x^3+8x^2+15x+18$. You can point out that when $x = 10$, we get just the answer we got above, 4968. Of course, isn't this just the place value idea? You can also point out that the polynomial arithmetic is actually easier than the integer arithmetic since there are no 'carries' to worry about.

The Puzzle. The numbers $2, 4, 5, 6, 8$ and 9 are arranged in a 3×3 multiplication table using each digit exactly once. It would look like the table below where each letter represents a different digit. The multiplication table is completed and the sum of the entries is tabulated. What is the largest possible sum obtainable? Before reading on, try this yourself. Even if you make only a little progress, you'll see how a productive stuggle can be for your students.

$\times$	a	b	c
d			
e			
f			

Solution: The sum of the six digits listed is 34. We can view the multiplication table as a rectangular array that is $A \times B$ where A is the sum of the entries along the top and B is the sum of the entries on the side. For example, if we use $a = 4, b = 5, c = 6, d = 2, e = 8$ and $f = 9$, then our rectangle would be $(4 + 5 + 6) \times (2 + 8 + 9) = 15 \times 19$, as shown.

	4	5	6
2	8	10	12
8	32	40	48
9	36	45	54

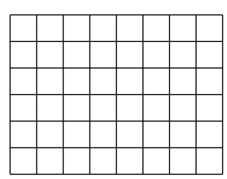
Now you can see that the sum of the entries in the multiplication table is nothing more that the area of the 15×19 rectangle, which is pretty quickly computed. In fact each rectangle you can build from these digits must have height plus width equal to 34. Therefore, if the width is $17 + x$, then the height is $17 - x$, as shown below. Looking at the area model to compute this product, we see that the area is $289 - x^2$.

	17	x
17	289	$17x$
$-x$	$-17x$	$-x^2$

The example above has $x = 2$, so the product is $289 - 4 = 285$. Now it is pretty clear that the largest possible sum is the largest possible area, and that is $17^2 - 0^2 = 289$. To be complete, one optimal solution is given below. Are there any other ways to achieve the target score of 289? Here's where the mystery comes in. You can ask your students to show you just their top row of numbers, the a, b and c. Then you compute $289 - (17 - (a+b+c))^2 = (a+b+c)(d+e+f)$, which is pretty straightforward because $17 - (a+b+c)$ is usually very small, but takes a little practice. So you can tell them their sum without even writing any numbers.

	4	5	8
2	8	10	16
6	24	30	48
9	36	45	72

A final challenge. Let's pose one final problem, one that connects with the area model in a surprising way. Consider the 6×8 grid of squares below:



1. How many squares bounded by gridlines of any size are there in the grid? Note that there are 48 unit squares.

2. How many rectangular regions are bounded by grid lines?

Can you see how to apply the area model to get the answer to question 2? Here's a hint. Build the multiplication table for the numbers 1 through 8 times the numbers 1 through 6, and ask yourself how many rectangular regions have as a lower right corner the lower right corner of that square?