Reflections of a Problem Editor

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November 10, 2012

Abstract

This is an article by Leon Bankoff which appeared in Pi Mu Epsilon journal, Volume 6, Number 3, Fall 1975
From Wikipedia:
Leon Bankoff (December 13, 1908 – February 16, 1997), born in New York City, New York, was an American dentist and mathematician.After a visit to the City College of New York, Bankoff studied dentistry at New York University. Later, he moved to Los Angeles, California, where he taught at the University of Southern California; while there, he completed his studies. He practiced over 60 years as a dentist in Beverly Hills. Among many of his patients were celebrities.
Along with Bankoff's interest in dentistry were the piano and the guitar. He spoke fluent in Esperanto, created artistic sculptures, and was interested in the progressive development of computer technology. Above all, he was a specialist in the mathematical world and highly respected as an expert in the field of flat geometry. Since the 1940s, he lectured and published many articles as a co-author. Bankoff collaborated with Paul Erdős in a mathematics paper and therefore has an Erdős number 1.
From 1968 to 1981, Bankoff was the editor of the **Problem Department of Pi Mu Epsilon Journals,** where he was responsible for the publication of some 300 top problems in the area of plane geometry, particularly Morley's trisector theorem, and the arbelos of Archimedes. Among his discoveries with the arbelos was the Bankoff circle, which is equal in area to Archimedes' twin circles.

1 Introduction

Telling is not teaching and listening is not learning. This terse truism summarizes the difficulties in communication so often encountered in mathematical education. Nevertheless properly directed telling and intelligently oriented listening are essential components of successful communication. The most effective way to measure the degree of such success is by appropriate testing of the student's problem solving ability.

Volumes can be and have been written on the importance of problem solving in the learning process and in the growth and development of mathematics.

History is replete with instances where entire new branches of the art and science of mathematics have sprung up as a consequence of the search for the solution of some challenging problem. A noteworthy example is the successful attack on the brachistochrone problem by the Bernoulli brothers and the role played by this solution in the birth of the Calculus of Variations. Another familiar example is the emergence of the mathematical theory of probability as an offshoot of problems considered by Pacioli, Cardan and Tartaglia and the arousal of interest by the discussions between Pascal and Fermat. Even to this day, mathematicians continue to indulge in the age-old pleasurable activity of milking one another's brains:

- through conversation or correspondence
- exchanging ideas
- collaborating on the solution of difficult and perplexing problems
- hurling and accepting challenges emanating either from their own gnawing inquisitiveness or from the frustrated curiosity of others
- building, forging, developing and inventing new tools and ingenious devices in the never-ending struggle for the establishment of mathematical order out of chaos.

In addition to the influence of private communication in the advancement of mathematical knowledge, it is important to recognize the tremendous impetus occasioned by the dissemination of provocative, non- routine problems by way of mathematical journals.

For the last three centuries, readers of periodicals that contained problem sections have been invited to submit solutions to proposed problems with the objective of competing with other solvers for the publication of what the editors later judged to be the "best" solution.

First came the reader's pride in his successful bout with the challenging problem; then came his natural desire to display the results of his cerebration; and finally his curiosity as to how his solution stacked up against those submitted by other solvers. It has always been the function of the editor to solicit and select proposals suitable for the particular vehicle concerned and to use his best judgment in choosing solutions for publication. This often becomes a soul-searing problem for the editor, as will be discussed later.

One of the earliest periodicals to feature a section on problems was the Ladies' *Diary* which first appeared in London in 1704. In 1841 the Ladies' *Diary* and the Gentleman's *Diary*, which made its debut in 1741, were united and published under the title of The Lady's and, Gentleman's Diary, which came to an end in 1871. For some unaccountable reason, the title of the Ladies' Diary was changed to the singular form when it combined with the Gentleman's Diary.

The treatment of proposals and their solutions in these and in several other British publications of that era became a model for the *Mathematical* Questions from the *Educational* Times, which had its inception in 1863 and continued uninterruptedly until 1918. The spirit of the problem departments of the British journals was picked up by various French publications such as *L'Enseignement*

Mathémtique and *Mathesis* (Belgium) and also by the early American journals, notably the *Mathematical' Visitor*, which was launched at Erie, Pennsylvania in 1878.

In his introductory editorial to Volume I, Number 1 of the *Mathematical Visitor*, Artemas Martin, editor and publisher, had this to say:

In England and Europe, periodical publications have contributed much to the diffusion of mathematical learning, and some of the greatest scientific characters of those countries commenced their mathematical career by solving the problems proposed in such works.

*It was stated nearly three-quarters of a century ago that the learned Dr. Hutton declared that the Ladies' Diary had produced more mathematicians in England than all the mathematical authors of that kingdom.*Similar publications have produced like *results in this country. Not a few of our ablest teachers and mathematicians were first inspired with a love of mathematical science by the problems and solutions published in the mathematical department of some unpretending periodical.*

A world-renowned periodical that can certainly be considered "unpretending" despite its high level of sophistication is The American Mathematical Monthly, which was founded originally as a show case for proposed and solved problems. An exhaustive historical and statistical treatment of the problem departments of this journal from 1894 to 1954 appeared in the *Otto Dunkel Memorial Problem Book*, published by the Mathematical Association of America in August 1957 in commemoration of that Journal's fiftieth anniversary. The author of that survey, Mr. Charles W. Trigg, Dean Emeritus of Los Angeles City College, and one of the better known and most prolific problemists of our day, has put together a most informative, interesting and entertaining article well worth the attention of all mathematicians, whether active problemists or not.

One of the striking characteristics of most problem departments is the high incidence of participation by eminent mathematicians as well as by the "man on the street" lover of mental gymnastics. As one browses through the pages of the *Lady's and Gentleman's Diary*, the *Mathematical Questions* from the *Educational Times* or the *American Mathematical Monthly*, to name a few, one is impressed to discover what an attraction problems have held for so many who have achieved great prominence in mathematics. It comes as a surprise, for example, to learn that W. G. Horner, of *Homer's Method* fame, solved what is now known as the Butterfly Problem in the 1815 volume of the *Gentleman's Diary*. The list of problemists who participated in the problem department of the *Educational Times* reads like a veritable *Who's Who* in British Mathematics from 1863 to 1918. Among the active solvers may be found the names of Cayley, Cremona, Clifford, Sylvester, Whitworth, Todhunter, Hadamard, Hardy, Salmon, Beltrami and countless others far too numerous to list. Currently the names of numerous prominent mathematicians may be found in the problem departments of the *American Mathematical Monthly*, the *Mathematics Magazine*, the *SIAM Review*, the *Pi Mu Epsilon Journal*, *Pentagon*, *School Science and Math-*

ematics, the *Journal of Recreational Mathematics*, the *Fibonaaci Quarterly*, the *Technology Review*, the *TWO-Year College Mathematics Journal* , *Elemente der Mathematik* (Switzerland), and the *Mathematics Student Journal*. It is hard to estimate how many high schools and two-year colleges publish "newsletters" primarily, for their own students. Examples are the *Indiana School Mathematics Journal* and the *Oklahoma University Mathematics Letter*. Others are listed in a booklet issued by the *National Council of Teachers of Mathematics*, authored by William L. Schaaf and entitled "*The High School Mathematics Library*".

On a less formal basis, practically every issue of Martin Gardner's *Mathematical Games Department* in the *Scientific American* offers several intriguing problems for the entertainment and enlightenment of its readers, with solutions revealed in the following issue. Some of these problems have been known to generate heated controversy and discussion, all to the betterment of mathematical science.

In addition to its noteworthy expository articles, the *Mathematical Gazette*, while not containing a problem department, does nevertheless publish short provocative notes that frequently set off a chain-reaction of readership discussion and development. Furthermore, the *Gazette* maintains a Problem Bureau which offers assistance in the solution of problems whose sources are known. From those standpoints, the publication is a problemist's delight.

Of course, there are many specialized journals that do not maintain problem sections but most of the well-known ones do. It is hard to imagine the dismal change in character that would descend on a journal if its problem department were suddenly to be abandoned.

2 Problems of a problem Editor.

After the foregoing prelude, let us now come home to our own *Pi Mu Epsilon Journal* and dwell a bit on what goes on behind the scenes in the conduct of the Problem Department. Let us also consider what can be done to improve the department and to provide more enthusiasm and enjoyment among our problem devotees and the readers in general.

Problems in a great variety of categories have appeared in the *Pi Mu Epsilon Journal* since the time of its first appearance in April 1949. The Fraternity, which started at the *University of Syracuse* in 1903 as a mathematics club, achieved the status of a full-fledged chartered organization shortly after the academic year 1914-15, but it was not until 1949 that the *Pi Mu Epsilon Journal* blossomed forth. In the first issue, Editor Ruth W. Stokes got the problem department off to a good start by publishing eleven proposals, five of which were her own and the other six solicited from accommodating friends. With the exception of the Fall 1957 issue, the problem section has appeared regularly in each issue and it has been only on rare occasions that the editor was faced with a shortage of suitable proposals to the point where he was compelled to raid his own files to maintain an acceptable balance and variety in the proposal department.

Considering the relatively small circulation of the *Pi Mu Epsilon Journal* compared to some of the larger periodicals, the ratio of participants in the problem department is rather high. However, it is quite likely that many of the readers solve the problems, file them away and never get around to submitting the solutions. Readers are urged to try their hand at problem composition and to offer their solutions for the possible publication. One never know when the presence of an unusual gimmick or a clever solution device might in itself warrant the publication of the solution.

This could be interpreted as a cry for help. The most difficult task for the problem editor is not the selection of solutions for publication but rather the selection of proposals of a type that elicits reader response. By soliciting contributions from a wider cross-section of the membership and from other interested readers, the editor hopes to achieve a diversity of high-quality proposals in geometry, analysis, number theory, inequalities, mathematical logic, game theory, set theory, group theory, probability, paradoxes fallacies and cryptarithms, to name a few. In general, problems should rise above the level of unimaginative textbook exercises and should strive to give solvers an opportunity to demonstrate ingenuity and inventiveness.

One of the essential attributes of a suitable proposal is the hard-to-define quality of elegance. This characteristic is usually associated more with solutions that with proposals but is nevertheless an important element in attracting the attention of would-be solvers. A beautiful example of an elegant proposal is the following one, due to W.
J. Blundon, of the Memorial University of Newfoundland:

Let I, O, H denote respectively the incenter, the circumcenter and the orthocenter of a triangle with sides a, b, c and inradius r. Prove that the area K of the triangle IOH is given by

$$K = \frac{|(a-b)(b-c)(c-a)|}{8r}.$$

This problem was proposed in January 1967 issue of *Elemente der Mathematik* and solution was published the following January. Opinions regarding beauty are often debatable but can anyone deny that the economy of expression in the displayed result constitute a pure and austere elegance? One would hope that a proposal of such high artistic merit would elicit a solution of comparable elegance.

Not all proposals can aspire to high level of elegance in their mere statement. Most problems are straightforward challenges to duplicate or improve upon results already found by the proposer, especially if the method of solution or the final result is significant, novel generalized, instructive or entertaining. Ordinarily, problem editors require solution submitted along with proposals. The purpose of this is to assist the editor in evaluation of the suitability of the proposal, the complexity of the solution or the expected readership response. On the other hand, conjectures and unsolved problems connected with related investigations or research projects are some times submitted with the hope that

someone may successfully arrive at a satisfactory solution. **When such proposals are published, the readers are alerted to the fact that solutions have not been provided**.

Since the *Pi Mu Epsilon Journal* appears only twice a year, acceptable proposals are filed away for possible use some time in the future. This may entail long delays in publications, especially if other problems in like categories have priority. Unused or unusable proposals will be returned upon request.

After an issue of the *Journal* comes off the presses and is sent to the subscribers, solutions begin to trickle in. In due course the contributions are acknowledged, the solutions are filed away and the envelope in which they were mailed are discarded. That is why solvers who would like to receive credit for their labors should be sure to identify their solutions with their names and addresses. Solutions to more than one problem should be sent on separate sheets and to facilitate filing, should not be co-mingled with extraneous correspondence. This saves the editor the inconvenience of photocopying portions for separate filing.

With the approach of deadlines for submitting the copy to the *Journal* Editor your problem editor examines all solutions received and is often confronted with difficult decisions as to which solution to publish. He is reminded of what motivates problemists to submit solutions in the first place. Why do they not simply solve it, file it and forget it ? One incentive, of course, is the altruistic desire to share with others a well-thought out and well-expressed solution; another is to gratify one's ego in a most acceptable way by seeing his creation appreciated and published. Some problemists are so well versed in so many diverse branches of mathematics that they breeze through most of the proposals with ease and take a delight in making a marathon game of their knack for prolixity. These are individuals who generally combine quality with quantity and in many cases are legitimate candidates for inclusion in the Guinness Book of Records. The frequency with which their solutions are published may lead other solvers to suspect favoritism on the part of the editor, but readers are hereby assured that every effort is made to select solutions objectively on the basis of merit.

When submitting a solution, the solver should try to present it in the format adopted by the problem department. This saves the editor time and trouble in re-typing it for the printer. Most problem editors are their own secretaries – unsung heroes who make a labor of love out of serving as intermediaries between proposers and solvers. Consequently, when they are confronted with a difficult choice between two otherwise excellent solutions they may just tip the scales in favor of the solution that permits them to follow the path of least resistance. On the other hand, neatness and good form cannot in themselves supersede content; while they are qualities that are greatly appreciated, editors are often grieved to have to turn down a solution despite the evidence of painstaking care in presentation.

On occasion, excellent solutions with widely separated approaches are found to be too good to be lost to posterity. In those cases a diligent editor will attempt to do justice by concocting an amalgam of the solutions or, if space permits, publishing multiple solutions. Here again, the best mathematical and

literary expression is considered along with the quality of the solution.

It may be hard to believe, but your problem editor occasionally receives an answer to a problem instead of a solution. Participants in this arena are not really concerned with answers; their primary interest is in the *way* the solution was found – the train of thought that led to the solution, the transparency of the solver's heuristic approach to the problem, essentially, the solver's ability to take the reader by the hand and literally lead him over the various steps of the proof. One of the tests of elegance is finding a way of doing this adroitly without insulting the reader's intelligence by spelling out procedures that should be evident to him. At the same time, the solution should avoid the sins of omission – skipping steps that are necessary for a full understanding of the solution, proof or construction, as the case may be.

To achieve this ability, the solver should be familiar with the criteria for elegance – what we call the **ABCD**'s of Elegance. They are **A** for Accuracy, **B** for Brevity, **C** for Clarity, and **D** for the Display of Insight, Ingenuity, Imagination, Originality and, where possible, Generalization. It always helps to be able to instill a dramatic sense of awe, wonder and surprise. These are the intangible qualities that elevate mathematical creations to the realm of high art, whether they be proposals, solutions, short notes, expository essays or chapters in some impressive tome. In conclusion, it is hoped that the enunciation of these high ideals will inspire readers to make efforts to achieve them without detering them from their most welcome participation in the Problem Department of this *Journal*.