Special Topics in Middle School Mathematics, Math 6050

This course is designed to enable middle school mathematics teachers to address the challenges posed by both the Practice and the Curriculum Common Core State Standards (see `http://www.corestandards.org/Math/`). The course will meet on 11 or 12 Saturday mornings for three hours each and on 2 teacher work days starting in October, 2015. Included among the non-credit participants are local MATHCOUNTS teams. Teachers taking the course for credit will be expected to attend the two teacher work day meetings and will have modest homework and lecturing assignments. We will use modifications of the papers listed below.

1. Place value and digit problems (nine hours). Place-value notation is at the heart of integer and fraction arithmetic. To become arithmetically fluent requires the clear understanding of the representation of integers and rational numbers. To establish this fluency, we study other methods of representation including fractional, negative and irrational bases, and the arithmetic to which these representations give rise.

2. Fractions (three to six hours). The gateway to algebra, fractions represent a major hurdle in middle school mathematics. Students who struggle with algebra usually do so because they lack fluency in arithmetic, and most often this boils down to failure to understand the arithmetic of fractions. Fractions (ie, rational numbers) are at the same time both points on the number line and distances from zero on a line. In addition fractions are ways to measure relationships. We take the CCSS approach that all fractions are integer multiples of *unit* fractions.

3. Sets and Counting (12 hours). It is not much of a stretch to say that there are three types of fundamental objects in mathematics: numbers, sets, and functions. The student of functions is usually left to algebra and calculus students. The naive (non-axiomatic) approach to sets is a natural topic at the middle school level. Building larger sets from given ones, determining set equality and the operations of union, intersections and complementation are fundamental ideas which continue to show up in later courses. The most interesting problems occur in the area of combinatorics of sets. That is, counting. This is where permutations and combinations come into play. Important ideas such as the Inclusion/exclusion principle and the Pigeon-hole principle play a big role in combinatorics. We will also discuss discrete probability. Some of the counting problems are accessible only after the discussion of divisors at the end of the course.

4. Combinatorial Games (three hours). This is a very enjoyable topic that does not require much technical knowhow. The games we discuss are Bouton's Nim, and several one-pile static and dynamic games, games with crippled rooks and queens, and other 'handicapped' chess pieces.

5. Logic (three hours). Here we study logic and its relationship with sets and algebra.

6. Number Theory (18 hours). The remarkable relationship between integers on one hand and polynomial functions on the other enables students fluent in arithmetic to thrive when learning polynomial arithmetic, which is an important part of algebra.This remarkable relationship begins with representations. Integers have multiple representations just as polynomials do, rangin from standard form (place value) to factored form. For example $36 = (5+1)(4+2) = 2^2 3^2$ shows the standard form, a hybrid form, and the factored form of the number thirty-six. Other topics in this area are modular arithmetic, the Hasse diagram for divisors, and the irrationality of square roots (from the Fundamental Theorem of Arithmetic).

References

Harold Reiter, http://math2.uncc.edu/ hbreiter/Epsilon/ePlacevalueProblems.pdf
Harold Reiter, http://math2.uncc.edu/ hbreiter/Epsilon/eFusingDots.pdf
Harold Reiter, http://math2.uncc.edu/ hbreiter/Epsilon/eDivisors.pdf
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