The Process Standards

NC Ready for Success

HAROLD B. REITER

UNIVERSITY OF NORTH CAROLINA

CHARLOTTE

http://math2.uncc.edu/~hbreiter/

First let me say how delighted I am to be here. Thank you Debbie Hill and Maggie Chotas for inviting me to speak at this fine event. As I begin my fifty-first year of teaching, I am more optimistic than I have been in many years about the changes that are taking place nationally in the teaching of pre-college mathematics. I have decided, by the way, to keep teaching until I figure out how to do it right. On the other hand, I am confident that we teachers of North Carolina are on the verge of making improvements that will enable our students eventually to compete internationally in STEM fields.

What do the new standards mean for us? Above all else, they encourage us to teach mathematics so that it makes sense to our students. No longer can we simply say 'do it this way, and don't worry about why it works.' Its also important that teachers have a sound understanding of where the topics they are teaching will show up again in the curriculum.

Here's a wonderful quote from second grade teacher Austen Kassinger "I know firsthand the intense anxiety that math assignments can provoke. But I'm certainly not sorry that they are being taught Common Core State Standards. As a math teacher, I have embraced these standards. I have seen kids solve problems I never would have thought to ask, then explain their thinking and justify their answers with pride. I have seen kids who used to hate the subject scribble, "**I love math!**" all over their tests. That joy is rooted in their persistence."

There are two major topics, among others, for which the CCSS approach is sounder pedagogically than what we have had before. These are *place-value* and *fractions*. The three problems below involve these topics. These problems are elementary, yet the ideas show up repeatedly later in the curriculum. The first deals with fractions, the second with integer multiplication, and the third with the area model for multiplication.

Let us take am minute to debunk a few myths about CCSSM. The standards do not dictate a curriculum or a method of teaching, they do not call for a particular assessment, they do not tell us how to evaluate teaching.

1. Name a fraction between 1/2 and 2/3. This is one of my favorite problems for seven and eight year-olds. The answers they give reveal a lot about their mathematical development. Please take a minute to settle on an answer before we discuss the most popular answers.

$$\frac{1}{2} < \frac{7}{12} < \frac{2}{3}$$

$$\frac{1}{2} < \frac{5}{9} < \frac{2}{3}$$

$$\frac{1}{2} < \frac{5}{8} < \frac{2}{3}$$

$$\frac{1}{2} < \frac{3}{5} < \frac{2}{3}$$

$$\frac{1}{2} < \frac{7}{12} < \frac{2}{3}$$

Of course this is the midpoint or the average of the two fractions, $\frac{1}{2}(\frac{1}{2} + \frac{2}{3}) = \frac{1}{2} \cdot \frac{14}{12} = \frac{7}{12}$

$$\frac{1}{2} < \frac{5}{9} < \frac{2}{3}$$

This is the answer I got from a six year old. That he nailed the question did not surprise me, but his explanation did. He said, $\frac{1}{2}$ is the same as $\frac{5}{10}$ which is less that $\frac{5}{9}$. And $\frac{5}{9}$ is less that $\frac{6}{9}$ which is the same as $\frac{2}{3}$.

$$\frac{1}{2} < \frac{5}{8} < \frac{2}{3}$$

Who provides this answer? A carpenter or anyone who works with rules graduated in eighths of an inch.

$$\frac{1}{2} < \frac{3}{5} < \frac{2}{3}$$

My favorite answer is obtained by thinking about the water-sharing model. A pair of bicyclers has a bottle of water between them. Three other cyclers have two bottles of water among them. If they all agree to meet and share equally, how much will each cycler get? This number $\frac{3}{5}$ is called the *mediant* of the two fractions. We see this answer a lot when fifth graders learn addition of fractions incorrectly. Let's use the water sharing model to help our fifth graders understand fraction addition and fraction comparisons. By the way, do you ever wonder why 10 year old boys can very easily compare the fractions $\frac{6}{18}$ and $\frac{7}{19}$. Baseball!

2.

$$23 \times 96 = 69 \times 32$$

$23 \times 96 = 69 \times 32$

When I present this problem to students, I do not say any more. I want them to decide what we should ask about the equation. For example, is it true? How do we verify its correctness. Please think about this a moment before we continue.

There are at least three ways to think about it. We can multiply it out to get both sides in standard (place value) form. We can factor both sides into primes and see that the factors match. Or we can write

$$23 \times 96 = 23 \times 3 \times 32 = 69 \times 32.$$

Children seem to like this third method better than adults.

The third problem requires some preliminary discussion. Consider the product 123×12.

	100	20	3
10	1000	200	30
2	200	40	6

Notice that this picture shows the distribution law at work. It also provides some insight into why we arrange the digits the way we do when we multiply using vertical format. What about 119×12?

	100	20	-1
10	1000	200	-10
2	200	40	-2

Finally we have the algebra problem 'Compute the product of the two polynomials $x^2 + 2x - 1$ and $2x + 1$

	x^2	$2x$	-1
x	x^3	$2x^2$	$-x$
2	$2x^2$	$4x$	-2

So the answer is $x^3 + 4x^2 + 3x - 2$. What do we take away from this example? One point is that if third grade teachers understand the important uses of the area model in teaching algebra, maybe they would spend a bit more time with the area model, and if that happened, 8th grade teachers would never have to use the f-word FOIL. Would that be great!

My friends Paul Zeitz and Sam Vandervelde repeatedly say that a math circle leader should never leave all the questions resolved. Give something for everyone to think about, they say. So I leave you with question 3. I have a few small prizes for the first folks who find a nice idea for solving it.

3 Distribute the numbers $1, 2, 3, 6, 7, 8$ in the six positions a, b, c, d, e, f so that when you build the multiplication table, the sum of the nine products $ad + ae + af + bd + \cdots + cf$ is as large as possible.

	a	b	c
d	ad	bd	cd
e	ae	be	ce
f	af	bf	cf

Solution: Added later: The best approach is to recognize that the sum of the nine numbers is the area of a rectangle with perimeter $2(1+2+3+6+7+8) = 54$. Since 54 is not a multiple of 4, and the rectangle must have integer sides, the best we can hope to do is to build a 13×14 rectangle. Indeed we can do this if we choose $2, 3, 8$ for one side and $1, 6, 7$ for the other side. In this case we get an area of 182. Can you prove that the rectangle which has the greatest area among those with a fixed perimeter is a square? No calculus needed here.