Using KenKen to Build Reasoning Skills

1 Introduction, Day 1

Ok, so here’s the first day’s discussion. Each day we’ll discuss another Tactic and then practice it. The topic for today is parity. Parity refers to the oddness or evenness of a number. In the case of KenKen, we are interested in the parity of the sum of the entries of a cage. For example, every cage with clue 7+ is odd because whatever numbers go in the cells of this cage, their sum is 7, which is odd. But a cage with clue 6× could be odd or even, depending on the number of cells in the cage. For example, a two cell cage with clue 6× either is {1, 6} or {2, 3}, odd either way. But if the cage had 3 cells, it would have {1, 1, 6} or {1, 2, 3}, so it would be even. You’ll see later in the week that parity is just a tip of the iceberg. We can use modulo 3 and modulo 4 arithmetic in some cases. Parity is just another way to say modulo 2 arithmetic. Besides reading this Day 1 discussion, your homework for tomorrow is the H puzzle.

2 Parity and fault lines

A fault line is a heavy line that cuts entirely through the puzzle. Fault lines often provide the opportunity to use parity or other ideas because they cut the puzzle into a smaller puzzle of manageable size. Parity refers to evenness and oddness of a cage. Specifically, the parity of a cage $C$ is even (odd) if the sum of the entries of the cage is an even (odd) number. For example, is an odd cage the sum of the entries is 11, which is an odd number. Some two-cell cages have determined parity even though the candidates are not determined. For example, is an even cage because the entries are either both even or both odd. On the other hand there are two-cell cages that can be either even or odd. For example, has two pairs of candidates, {2, 6} and {3, 4}.

So how can we use parity to make progress towards a solution? Consider the row from a $6 \times 6$ KenKen:

Because the sum $1 + 2 + 3 + 4 + 5 + 6 = 21$, the row must have exactly one or exactly three odd cages. Since the two [1−] cages are odd, so must be the [12×] cage. There is another way to look at the problem of determining the candidates
for the $[12\times]$ cage. If we put 2 and 6 is the $[12\times]$ cage, where would the 1 go. Since 1 can go only with 2 in a $[1-]$ cage, the $\{2, 6\}$ cannot be the set for the $[12\times]$ cage. But consider the two-row KenKen fragment below.

The set of the $[12\times]$ cage is one of $\{1, 3, 4\}$, $\{1, 2, 6\}$, or $\{2, 2, 3\}$, the last two of which are odd. But the two cages $[3\div]$ and $[10+]$ are even and the two cages $[1-]$ are both odd. The sum of the entries in the two rows is 42, so the number of odd cages must be even. Therefore the $[12\times]$ cage can have only the digits $1, 3, \text{ and } 4$.

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Of course we can sometimes use parity when there are no fault lines. Consider the puzzle part below:
Notice that all three of the cages $[18\times], [6+]$ and $[12+]$ are even cages while $[15\times]$ is an odd cage. Therefore the entry in the top cell of the $[11+]$ cage must be odd. One (non-unique) solution is

\begin{tabular}{|c|c|c|c|}
\hline
12+ & 4 & 2 & 3 \\
\hline
11+ & 5 & 6 & 1 \\
\hline
15\times & 6+ & 6 & 5 & 1 \\
\hline
\end{tabular}