1 Introduction, Day 3

Some puzzles have a row or a column all the cells of which belong to just a few cages. In other words, each cage that has a cell in the column (row) does not have any cells in any other column. In that case there are often implications about the possible candidates for the cage. We’ll also discuss the (sudoku-named) strategy called X-wing. Besides reading this Day 3 discussion, your homework for tomorrow is the L puzzle. It will be interesting to get reports of where your breakthrough occurred.

2 Stacked Cages

Some puzzles have two or more cages confined to a single line (a row or a column). In this case, we call the cages stacked, and we can often take advantage of this situation. Consider the fragment below.

Parity does not help much. All we know from parity is that \( x \) is even. This follows from the fact that \([24\times]\) is odd (it’s either \( \{1, 4, 6\} \) or \( \{2, 3, 4\} \)) and \([2-]\) is even as we saw above. Since the sum of each line in a 6 × 6 puzzle is 21, the entry \( x \) must be even. But we can learn more as follows. The cage \([24\times]\) contains the 4 of its row. Therefore, the \([2-]\) cage does not contain 4, from which it follows that \( 3 \in \{[2-]\} \). But in this case, it now follows that \( \{[24\times]\} = \{1, 4, 6\} \). Now we can see that \( \{[2-]\} = \{3, 5\} \), and from this it follows that \( x = 2 \). We’ve used some new notation here. The notation \( \{a\} \) means the collection of candidate sets for the cage \( a \). Note that we used the word collection, not set, because sets are not allowed to have repeated members, but a cage can have a digit more than once (in different rows and columns. This kind of object that looks like a set, but can have multiple membership is sometimes called a multiset. For example, \( \{1, 1, 1, 2, 2\} \) is a five element multiset, but as a set would have just two elements, 1 and 2. For example consider an \( L \)-shaped three cell cage in a 6 × 6 puzzle with the clue \( 12\times \). Then \( \{[12\times]\} = \{\{2, 2, 3\}, \{1, 3, 4\}, \{1, 2, 6\}\} \). Notice that it is a very complicated...
object, a set whose members are multisets. See what inferences you can draw in each of the following cases.

1. \[ 2 \div 2 \div 6 + \]

2. \[ 2 \div 2 \div 1 - \]

3. \[ 2 - 2 - 6 \times \]

4. \[ 2 \div 3 \div x, y \quad x, y \]

3 The X-wing strategy

The X-wing strategy refers to the fact that no \( k \) parallel lines can have more than \( k \) copies of a given symbol. In the sample case below, we use the fact that there are at most two 2's in the two rows, and then use parity and counting to finish the problem. Find the candidate sets for each cage before looking at the solution on the next page.

\[
\begin{array}{cccc}
18 \times & 12 \times & 1- & 15 \times \\
2 \div \\
\end{array}
\]
The candidate multisets for \([15 \times]\) and \([18 \times]\) all contain 3, so the cage \([12 \times]\) cannot contain a 3. Therefore \(\{12 \times\} = \{2, 6\}\). Now the 4 in the top row must be in the \([1-]\) cage, and it cannot go with a 3 so \(\{1-\} = \{4, 5\}\). The rest is straightforward.

<table>
<thead>
<tr>
<th>18×</th>
<th>12×</th>
<th>1−</th>
<th>15×</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3</td>
<td>2, 6</td>
<td>2, 6</td>
<td>4, 5</td>
</tr>
<tr>
<td>1, 3, 6</td>
<td>2÷</td>
<td>4, 5</td>
<td>1, 3, 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15×</th>
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