Information for Teachers

The Julia Robinson Math and Computing Festival day prize problems for grade six students on the next few pages are designed for your female students. They are hard problems that your students might find interesting. If they do find them interesting, the problems have accomplished their purpose. Students do not have to work these problems to be successful. In fact, we would like you to pick a prize-winning student even if your students do not solve these problems. We hope you will register your prize winner at the website http://education.uncc.edu/oeo/jrfm/, and then urge them to attend. You can ask them which of the prizes they would like, a book prize or a t-shirt prize and you can let us know by means of the online web registration. At that time we hope you will agree to help us that day March 26, 2011. Only female middle school students are invited to take part that day. Plans are to discuss these problems at the December 18th meeting of the Charlotte Teachers’ Circle, and again at a later date.
1. **The 1 through 6 problem.** Take the digits 1 through 6 in order, insert arithmetic operations and parentheses, and build the numbers from 1 to 20. For example, we can write 1 as \((1 + 2) + (3 - 4)) + (5 - 6)\). Be sure that your answer is parenthesized enough to make it unambiguous. For example, \(1 - 2 - 3 - 4 + 5 - 6\) can be interpreted in several ways, so it would not be an acceptable answer.

2. (This problem is due to J. Parker Garrison.) The science lab has just purchased some new sea creatures for the new aquarium. An assortment of octopi, clown fish and seahorses were placed in the tank. There were a total of 32 tentacles, 20 fins, and 13 heads. How many of each creature was placed in the tank? (NOTE: Clown fish have 2 fins and 1 tail, but seahorses have 3 fins and 1 tail, and yes, octopi have a head!)

3. A watermelon weighs 100 pounds. It is known that it is 99% water (by weight). After a month of dehydration, it is found to be just 98% water. How much does it weigh?

4. **A Big Block Problem.** A rectangular block of size \(5 \times 6 \times 7\) is built from 210 unit cubes. How many of the 210 cubes can be seen from the outside? How many are visible when you look at the cube from a corner (where you can see three adjacent faces all meeting at that corner)?

5. (This problem is due to Nicholas Miklaucic.) A burger, a frie and 2 sodas cost $2.49. A burger, two fries, and a soda cost $3.00. Two burgers, a frie, and a soda cost $3.59. What is the cost of a soda, a burger, and a frie?

6. **Neat Fractions.** Find four different digits selected from the set \{1, 2, 3, 4, 5, 6, 7, 8, 9\} to build two fractions each with a single digit numerator and single digit denominator so that the sum of the two fractions is less than 2 but as large as possible otherwise. Can you prove that your fraction is as large as it can be? You might get started on this by asking the question about a single fraction \(\frac{a}{b}\). You’re looking for the \(a\) and \(b\) so that \(\frac{a}{b}\) is as large as possible but less than 2. Why is \(\frac{9}{5}\) a better choice than \(\frac{5}{4}\)?
7. The 8 × 10 grid below has numbers in half the squares. These numbers indicate the number of mines among the squares without numbers. Use this information to find the exact location of all the mines.

For example, if you were given

\[
\begin{array}{ccc}
1 & 1 & \\
2 & 2 & \\
1 & 1 & \\
\end{array}
\]

you’d find that there are 3 mines as shown:

\[
\begin{array}{ccc}
\bullet & \bullet & \\
\bullet & \bullet & \\
\bullet & \bullet & \\
\end{array}
\]

8. How many triangles of all sizes are there in the picture?
9. We are allowed to insert one of the signs \{+,-,\times,\div\} in each gap in the string \(7\ 19\ 6\ 5\ 8\ 5\) together with parentheses to achieve an expression whose value is exactly 100. For example, \(7 \times (19+(6-5)) - (8 \times 5) = 140 - 40 = 100\). For each example below, build an expression whose value is exactly 100.

(a) \[\underline{6} \underline{3} \underline{5} \underline{7} \underline{5}\]

(b) \[\underline{8} \underline{4} \underline{2} \underline{5} \underline{5} \underline{5} \underline{5}\]

(c) \[\underline{7} \underline{7} \underline{7} \underline{7} \underline{2}\]

(d) \[\underline{9} \underline{8} \underline{7} \underline{6} \underline{5} \underline{4}\]

(e) \[\underline{4} \underline{7} \underline{5} \underline{5} \underline{9} \underline{1}\]

10. Elizabeth has tiles of three types, \(1 \times 1\) : \[\square\], \(2 \times 2\) : \[\boxed{}\], and \(3 \times 3\) : \[\boxed{\boxed{\boxed{\boxed{}}}\}\]. For each of the boards listed, find the fewest tiles required to cover the board without any overlap.

(a) \(4 \times 4\)
(b) \(5 \times 5\)
(c) \(6 \times 6\)
(d) \(7 \times 7\)
(e) \(8 \times 8\)
(f) \(9 \times 9\)
(g) \(10 \times 10\)

11. (This problem is due to Parker Garrison) If one grasshopper can jump as high as 5 frogs, and 2 fleas can jump as high as 7 grasshoppers, how many frogs can jump as high as 8 fleas?