Information for Teachers

The Julia Robinson Math and Computing Festival day prize problems for grade eight students on the next few pages are designed for your female students. They are hard problems that your students might find interesting. If they do find them interesting, the problems have accomplished their purpose. Students do not have to work these problems to be successful. In fact, we would like you to pick a prize-winning student even if your students do not solve these problems. We hope you will register your prize winner at the website http://education.uncc.edu/oee/jrmf/, and then urge them to attend. You can ask them which of the prizes they would like, a book prize or a t-shirt prize and you can let us know by means of the online web registration. At that time we hope you will agree to help us that day March 26, 2011. Only female middle school students are invited to take part that day. Plans are to discuss these problems at the December 18th meeting of the Charlotte Teachers’ Circle, and again at a later date.
1. **The 1 through 6 problem.** Take the digits 1 through 6 in order, insert arithmetic operations and parentheses, and build the numbers from 1 to 20. For example, we can write 1 as \(( (1 + 2) + (3 - 4) ) + (5 - 6) \). Be sure that your answer is parenthesized enough to make it unambiguous. For example, \(1 - 2 - 3 - 4 + 5 - 6 \) can be interpreted in several ways, so it would not be an acceptable answer.

2. **A Big Block Problem.** A rectangular block of size \(5 \times 6 \times 7\) is built from 210 unit cubes. How many of the 210 cubes can be seen from the outside? How many are visible when you look at the cube from a corner (where you can see three adjacent faces all meeting at that corner)?

3. **Neat Fractions.** Find four different digits selected from the set \(\{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) to build two fractions each with a single digit numerator and single digit denominator so that the sum of the two fractions is less than 2 but as large as possible otherwise. Can you prove that your fraction is as large as it can be? You might get started on this by asking the question about a single fraction \(\frac{a}{b}\). You’re looking for the \(a\) and \(b\) so that \(\frac{a}{b}\) is as large as possible but less than 2. Why is \(\frac{9}{5}\) a better choice than \(\frac{7}{4}\)?

4. Let \(N\) be the huge number

\[
N = 13579111315 \ldots 1999
\]

obtained by writing down, in order, the representation of the first 1000 odd positive integers.

(a) How many digits does \(N\) have?
(b) How many times does the digit 6 appear in \(N\)?
(c) What is the product of the 2009th digit and the 2010th digit of \(N\)?
5. We are allowed to insert one of the signs \{+,-,\times,\div\} in each gap in the string 7 19 6 5 8 5 together with parentheses to achieve an expression whose value is exactly 100. For example, \(7 \times (19 + (6 - 5)) - (8 \times 5) = 140 - 40 = 100\). For each example below, build an expression whose value is exactly 100.

(a) 
6 3 5 7 5

(b) 
8 4 2 5 5 5 5

(c) 
7 7 7 7 2

(d) 
9 8 7 6 5 4

(e) 
4 7 5 5 9 1

6. How many triangles of all sizes are there in the picture?

7. (This problem is due to Nicholas Miklaucic.) A burger, a frie and 2 sodas cost $2.49. A burger, two fries, and a soda cost $3.00. Two burgers, a frie, and a soda cost $3.59. What is the cost of a soda, a burger, and a frie?
8. This problem is about using cubes to build polyhedra that resemble buildings. Students can practice spacial visualization and use their imagination. Here’s a sample problem. Find all possible cubical buildings that have a base, and front projection and a right side projection that is \[
\begin{array}{cccc}
\hline \\
& & & \\
& & & \\
\end{array}
\]

**Solution.** There are seven solutions. We depict them using the base diagram where the number in each square represents the number of cubes on top of that square. For example \[
\begin{array}{cc}
2 & 2 \\
2 & 2 \\
\end{array}
\]
is the solution that uses the maximum number of cubes, whereas \[
\begin{array}{ccc}
1 & 2 & 1 \\
2 & & \\
\end{array}
\]
uses the minimum number of cubes. How many solutions are there altogether? Answer: 7. Two solutions can be built with 6 cubes, four with 7 cubes and one with 8 cubes.

For each problem below, build a model that is consistent with the given perspectives.

(a) Is the number of cubes required determined by the three perspectives? Or, alternatively, are there several ways to build a model with these perspectives.

\[
\begin{array}{cccc}
\hline \\
& & & \\
& & & \\
\end{array}
\]

Top view

\[
\begin{array}{ccc}
& & \\
& & \\
\end{array}
\]

Front view

\[
\begin{array}{ccc}
& & \\
& & \\
\end{array}
\]

Right view

(b) Is the number of cubes required determined by the three perspectives?

\[
\begin{array}{cc}
& \\
& \\
& \\
\end{array}
\]

Top view

\[
\begin{array}{cc}
& \\
& \\
\end{array}
\]

Front view

\[
\begin{array}{cc}
& \\
& \\
\end{array}
\]

Right view

(c) Is the number of cubes required determined by the three perspectives? What are the maximum number and minimum number of cubes needed to build the model? Build a model for it.
(d) The top and front projections are given. Build a possible right view. How many possible right views are there?

(e) Use exactly 20 cubes to make a model from the building plans below. Record the base plan for your building. What are the maximum and minimum numbers of cubes that could be used to build the structure.

9. Elizabeth has tiles of three types, $1 \times 1$, $2 \times 2$, and $3 \times 3$. For each of the boards listed, find the fewest tiles required to cover the board without any overlap.

(a) $4 \times 4$
(b) $5 \times 5$
(c) $6 \times 6$
(d) $7 \times 7$
(e) $8 \times 8$
(f) $9 \times 9$
(g) $10 \times 10$
10. The 8 \times 10 grid below has numbers in half the squares. These numbers indicate the number of mines among the squares that share an edge with the given one. Use this information to find the exact location of all the mines. A square can have more than one mine.

For example, if you were given \[
\begin{array}{cc}
1 & 1 \\
2 & 2 \\
\end{array}
\]
you’d find that there are 3 mines as shown: \[
\begin{array}{ccc}
* & * & * \\
\end{array}
\]

\[
\begin{array}{ccccccc}
1 & 1 & 2 & 2 & 2 & 2 \\
1 & 2 & 3 & 2 & 4 & \\
4 & 2 & 4 & 2 & 3 & \\
2 & 4 & 3 & 2 & 3 & \\
4 & 2 & 2 & 1 & 2 & \\
2 & 4 & 1 & 2 & 2 & \\
3 & 3 & 3 & 4 & 1 & \\
1 & 2 & 2 & 4 & 2 & \\
\end{array}
\]

11. In the series, \[S = 100 + 111 + 122 + \cdots + 1200,\] consecutive terms differ by 11. What is the sum?