1. Consider the game $G_1$ which starts with one pile of 10 counters. The rules allow a player to take 1, 3, or 5 counters on each turn. The player who makes the last move wins. Denote this game by $N(10; 1, 3, 5)$. Construct the directed graph version of this game. It has 11 vertices labeled 0 through 10, and a directed edge from a vertex $u$ to a vertex $v$ if and only if $u - 1 = v$, $u - 3 = v$, or $u - 5 = v$. Show that you can partition the 11 vertices into two groups $S$ and $U$ so that each edge from a member of $S$ goes to one in $U$, and some edge from each member of $U$ goes to one in $S$, and the vertex labeled 0 belongs to $S$.

2. Consider the game $G_2$ which starts with one pile of 20 counters. The rules allow a player to take 1, 2, or 5 counters on each turn. Denote this game by $N(20; 1, 2, 5)$. Do the same analysis with this game.