1. On the table there are 25 counters. Two people Al and Betty alternate removing counters. Al goes first. Each player can take from 1 to 4 counters on his turn. The person who picks up the last counter loses. Play this a few times with a partner.

We can denote this game with the notation $N'_4(25)$. The 25 is the size of the initial pile, the subscript 4 denotes the size of largest move, which remains constant through the game, and the prime $'$ means that we are playing the misère version of the game. In this case, that means last player loses. In the normal version of the game, the last player wins.

2. Two players take turns breaking up a 6 square by 8 square rectangular chocolate bar. They break the bar only at the divisions between the squares. If the bar breaks into several pieces, they keep breaking one piece at a time until only the squares remain. The first player who cannot make a break is the loser. Who will win?

3. Consider the game $G_2$ which starts with one pile of 20 counters. The rules allow a player to take 1, 2, or 5 counters on each turn. Denote this game by $N(20;1,2,5)$.

4. Recall the subtraction game in which two players start with two positive integers $a$ and $b$ written on a board. The first player subtracts one of the numbers on the board from a larger one, and write down the new difference. At each stage, the next player finds a positive difference between two numbers that is not already written on the board and writes it on the board. The first player who cannot find a new positive difference loses. For each of the pairs listed below, write down all the numbers that will eventually appear on the board, and use this information to state whether the game will be won by the first player or the second.

   (a) 35 and 42
   (b) 36 and 42
   (c) 39 and 42
   (d) 40 and 42

5. Next, we play a three-person game. The games is presented in so-called characteristic function form. What this means is that each coalition $C$ has a value $v(C)$ subject to conditions
(a) \( v(\emptyset) = 0 \)

(b) If \( A \subset B \), the \( v(A) \leq v(B) \).

Consider the game with three players, \( A, B, C \) such that \( v(\emptyset) = v(A) = v(B) = v(C) = 0, v(A, B) = 40, v(A, C) = 50, v(B, C) = 70 \) and \( v(A, B, C) = 100 \). In fully cooperative games players act efficiently when they form a single coalition, the grand coalition. Here the grand coalition is \( \{A, B, C\} \). The focus of the game is to find acceptable distributions of the payoff of the grand coalition. Distributions where a player receives less than it could obtain on its own, without cooperating with anyone else, are unacceptable - a condition known as individual rationality. Imputations are distributions that are efficient and are individually rational. Of course in this game all triplets \((a, b, c)\) of non-negative numbers satisfying \( a + b + c = 100 \) are imputations. We seek to find all of them that satisfy, in addition, \( a + b \geq 40, a + c \geq 50 \) and \( b + c \geq 70 \). For example, \((20, 30, 50)\) is one such imputation.

6. There are five rational pirates, \( A, B, C, D \) and \( E \). They find 100 gold coins. They must decide how to distribute them.

The pirates have a strict order of seniority: \( A \) is superior to \( B \), who is superior to \( C \), who is superior to \( D \), who is superior to \( E \).

The pirate world’s rules of distribution are thus: that the most senior pirate should propose a distribution of coins. The pirates, including the proposer, then vote on whether to accept this distribution. If the proposed allocation is approved by a majority or a tie vote, it happens. If not, the proposer is thrown overboard from the pirate ship and dies, and the next most senior pirate makes a new proposal to begin the system again.

Pirates base their decisions on three factors. First of all, each pirate wants to survive. Secondly, each pirate wants to maximize the number of gold coins he receives. Thirdly, each pirate would prefer to throw another overboard, if all other results would otherwise be equal.