March 26, 1999

The first 15 problems count 6 points each and the final one counts 20 points. Problems 1 through 12 are multiple choice and 13 through 15 are fill-in-the-blank. In the multiple choice section, circle the correct choice(s). You do not need to show your work on problems 1 through 15.

1. Which of the following numbers belong to the domain of the function \( h(x) = \sqrt{6 + x - x^2} \)? Circle all those that apply.
   
   (A) \(-2\)    (B) 0    (C) 2    (D) 3    (E) 5

2. The circle whose equation is \( 4x^2 + 4y^2 - 4x + 9y + 1 = 0 \) has a radius of
   
   (A) \(9/8\)    (B) \(9/4\)    (C) \(81/16\)    (D) \(81/64\)    (E) \(9/2\)

3. The graph of \( f(x) = ax^2 + bx + c \) is shown for certain values of \( a, b, \) and \( c \). Which of the following quantities are positive? Hint: \( f(1) = a \cdot 1^2 + b \cdot 1 + c \).
   Circle all that apply.
   
   \[
   \begin{array}{cccc}
   & & & \\
   & 4 & 3 & 2 & 1 & 0 & -1 & -2 & -3 & -4 \\
   & -4 & -3 & -2 & -1 & & & & & \\
   \end{array}
   \]

   (A) \(a\)    (B) \(c\)    (C) \(b^2 - 4ac\)    (D) \(a + b + c\)    (E) \(9a + 3b + c\)

4. Suppose \( g \) is defined by \( g(x) = \frac{4 - x}{3} \). Let \( f \) be the inverse of the function \( g \).
   Then \( f(2) = \)
   
   (A) \(-16\)    (B) \(-2\)    (C) \(1/3\)    (D) 2    (E) 8
5. If the domain of the function \( f(x) = 2x^2 + 5 \) is the interval \([-2, 1]\), which of the following numbers belongs to the range? Circle all those that apply.

(A) -2  (B) 0  (C) 5  (D) 12  (E) 15

The next four questions refer to the functions \( f \) and \( g \). Let

\[
f(x) = 2x - 3 \quad \text{and} \quad g(x) = \begin{cases} |x| & \text{if } x < 3 \\ x^2 - 1 & \text{if } x \geq 3 \end{cases}
\]

6. What is \( g \circ f(-2) \)?

(A) 1  (B) 4  (C) 7  (D) 15  (E) 48

7. Compute \((f/g)(-1) + (f \cdot g)(3)\).

(A) \(-4\frac{5}{8}\)  (B) -3  (C) 8  (D) \(5\frac{3}{8}\)  (E) 19

8. Find a value of \( x \) for which \( g \circ f(x) = 0 \).

(A) -6  (B) -5  (C) 0  (D) 3/2  (E) 6

9. Suppose \( 3 < x < 4 \). Which of the expressions describes the value of \( f(g(x)) \)?

(A) \(2|x|-3\)  (B) \(2x-3\)  (C) \(x^2-1\)  (D) \((2x-3)^2-1\)  (E) \(2x^2-5\)
The next three questions apply to the table given below: Suppose the functions \( f \) and \( g \) are given completely by the table of values shown.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>7</td>
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<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

10. What is the value of \( f(g(f(2))) \)?

(A) 0  (B) 1  (C) 2  (D) 6  (E) 7

11. Solve the equation \( g(f(x)) = 7 \) for \( x \)

(A) 1  (B) 3  (C) 4  (D) 6  (E) 7

12. What is \( f(g(1 + 2) + f(6 - f(4))) \)?

(A) 1  (B) 3  (C) 4  (D) 5  (E) 7

The next few questions are short answer questions. Write your answer in the blank provided.

13. Solve for \( x \): \( \sqrt{3 + x} = 3\sqrt{x} \).

14. A \( 4 \times 4 \times 4 \) cube is build from unit cubes. The entire outside surface is painted. How many of the 64 unit cubes receive some paint?

15. Suppose the curve \( y = 3x^2 + bx + 3 \) has exactly one \( x \) intercept. Find a value of \( b \)?
Work only one of the following two problems. If you show work on both, scratch out the work on the one you don’t want counted. **Show your work.**

16. Use the ‘completing the square’ technique to find the standard form of the quadratic function $y = x^2 + bx + c$ in terms of $b$ and $c$. Then find the vertex of the parabola.

17. Find all $x$ satisfying the inequality \( \frac{(x-1)(x+3)(x+6)}{(x+2)} \geq 0 \).