April 28, 1997
This test is white, your TA’s name is Tim and your lab section is ____________
Your name __________________________

The first 14 problems count 6 points each and the final 3 count 44 points as indicated. Total possible score 128. Because of this, you need not feel compelled to work every problem.

1. How many of the last eight lectures did you attend? There have been eight lectures since the last test. Please answer truthfully. Every answer will be counted as correct.
   (A) fewer than 3   (B) 3 or 4   (C) 5 or 6   (D) 7   (E) 8

2. The line \(y = -x + 29\) intersects the parabola \(y = 2x^2 - 3x + 5\) in two points. What is the midpoint of the line segment connecting those two points?
   (A) (0, 29)   (B) (1, 29)   (C) (1.5, 23.5)   (D) (0.5, 29.5)   (E) (0.5, 28.5)

3. Suppose \(\log_{10} x = 3.3003\). What is integer is closest to \(x\)?
   (A) 1900   (B) 1997   (C) 3,300.3   (D) 4.217   (E) 4.218

4. An \(5 \times 5 \times 5\) cube is made from 125 unit cubes. How many of these 125 unit cubes are exposed to the outside?
   (A) 90   (B) 98   (C) 100   (D) 110   (E) 125

5. The range of the function \(f(x) = x^2 + 4x + 5\) is the set of all numbers greater than or equal some number \(a\). Notice that 10 belongs to the range because \(f(1) = 1^2 + 4 \cdot 1 + 5 = 10\). What is \(a\)?
   (A) −3   (B) −2   (C) 0   (D) 1   (E) 3

6. Given that \(\log_b 2 \approx 0.356, \log_b 3 \approx 0.566, \) and \(\log_b 5 \approx 0.827\), which of the following is closest to \(\log_b 15000\)?
   (A) 3.93   (B) 4.41   (C) 4.62   (D) 4.76   (E) 4.95

7. A card is randomly selected from a deck of playing cards. Without replacing it, another card is selected. Which of the numbers below is closest to the probability that both cards are aces?
   (A) 0.0045   (B) 0.0054   (C) 0.0061   (D) 0.0066   (E) 0.0071
8. The rectangle $PQRS$ is divided into a square and three rectangles as shown. The area of the square is $x^2$ cm$^2$, and of the rectangles, $5x$ cm$^2$ and $3x$ cm$^2$, as shown. What is the area, in square centimeters, of the shaded rectangle?

(A) 15  (B) $15x^2$  (C) $8x^2$  (D) $15x$  (E) $8x$

9. The sum of three numbers $a$, $b$, and $c$ is 88. The same number results if we decrease $a$ by 5 or if we increase $b$ by 5 or if we multiply $c$ by 5. What is the value of $b$?

(A) 21  (B) 25  (C) 27  (D) 35  (E) 41

10. Find a quadratic function $y = ax^2 + bx + c$ which passes through the points $(-2, 2)$, $(0, -1)$ and $(2, 3)$. What is its value at $x = 4$?

(A) 14  (B) 16  (C) 17  (D) 18  (E) 23

11. Rounding off to the nearest half year, how long does it take an investment at 8% annual rate and compounded semi-annually (that is, twice a year) to triple?

(A) 13  (B) 13.5  (C) 14  (D) 14.5  (E) 15

12. A population of mice is found to decline each year by 10%. The initial population is 470. To the nearest mouse, how many mice are left after three years?

(A) 329  (B) 332  (C) 338  (D) 340  (E) 343

13. The sum of the coefficients of $x^3 - 3x^2 + 4x - 5$ is $1 - 3 + 4 - 5 = -3$. What is the sum of the coefficients of (the expanded form of) the polynomial $(2x - y)^3$?

(A) $-1$  (B) 1  (C) 8  (D) 27  (E) 64

14. A drawer contains exactly eight socks—two are red, two are green, two are yellow and two are blue. If two socks are selected at random without replacement, what is the probability that they match?

(A) $\frac{1}{7}$  (B) $\frac{7}{56}$  (C) $\frac{5}{28}$  (D) $\frac{2}{7}$  (E) $\frac{1}{3}$
On all the following questions, show your work.

15. (20 points) Let $S = \{1, 2, 3, 4, 5, 6\}$.
   
   (a) How many 3-element subsets does $S$ have?
   
   (b) Using members of $S$ as digits, how many 3-digit numbers can be made if repetition of digits is not allowed?
   
   (c) Using members of $S$ as digits, how many 3-digit numbers can be made if repetition of digits is allowed?
   
   (d) Using members of $S$ as digits, how many even 3-digit numbers can be made if repetition of digits is not allowed?

16. (12 points) Given that $x$ is one fourth of $y$ and $y$ is four greater than $x$, what is $x + y$?

17. (12 points) Find all values of $x$ for which $5 + \ln 2x = 4$. 