April 2000

Problems count 5 points each.

1. Find equations for all vertical and horizontal asymptotes of the function

\[ R(x) = \frac{x^4 + x^2 - 6}{2x^4 - 54x}. \]

**Solution:** Factor both numerator and denominator, then apply the asymptote theorem to get \( y = \frac{1}{2} \) as the horizontal asymptote, and \( x = 0, x = 3 \) for vertical asymptotes.

2. Solve the equation \( 3(2x - 5(x - 1) + 7) = 15 \).

**Solution:** \( x = 0 \) is the only solution.

3. Solve \( x - \frac{3}{x} = 6 \).

**Solution:** Use the quadratic formula on \( x^2 - 6x - 3 = 0 \) to get \( x = \frac{6 \pm \sqrt{36 + 12}}{2} = 3 \pm 2\sqrt{3} \).

4. What is the smallest root of \( 2x^3 + x^2 - 7x = 0 \)?

**Solution:** Factor to get \( x(2x^2 + x - 7) = 0 \) and then use the quadratic formula on the quadratic to find the smallest root to be \( -\frac{1 - \sqrt{33}}{4} \).
5. Let 

\[ f(x) = \begin{cases} 
2x - 1 & \text{if } x < 0 \\
 x + 3 & \text{if } 0 \leq x 
\end{cases} \quad \text{and } g(x) = |x| - 4 \]

Find a symbolic representation of \( g \circ f(x) \).

6. Solve 

\(-3 < 2x - 1 < 5\).

**Solution:** Add 1 to all three parts to get 

\(-3 + 1 < 2x < 6\) from which it follows that 

\(-1 < x < 3\). Writing this in one, \(-1 < x < 3\).
7. Find the vertex and sketch the graph of \( y = x^2 + 2x + 4 \).

**Solution:** Complete the square to get \( y = (x + 1)^2 + 3 \) which has vertex at \((-1, 3)\).

8. What is the domain of

\[
f(x) = \frac{\sqrt{x^2 - 1}}{x + 4}
\]
9. Find the center and the radius of the circle given by
\[ x^2 + 6x + y^2 - 4y = -4. \]

**Solution:** The center is \((-3, 2)\) and the radius is 3.

10. What is the midpoint of the line segment from \((3, -6)\) to \((5, 12)\)?

**Solution:** use the formula to get \(((3 + 5)/2, (-6 + 12)/2) = (4, 3)\).

11. Suppose \(g\) is defined by \(g(x) = (4 - x)/6\). Let \(f\) be the inverse of the function \(g\). Find a symbolic representation of \(f\).

**Solution:** \(y = -6x + 4\).