April 2000

Problems count 5 points each.

1. Find equations for all vertical and horizontal asymptotes of the function

\[ R(x) = \frac{x^4 + x^2 - 6}{2x^4 - 54x}. \]

2. Solve the equation \( 3(2x - 5(x - 1) + 7) = 15 \).

3. Solve \( x - \frac{3}{x} = 6 \).

4. What is the smallest root of \( 2x^3 + x^2 - 7x = 0 \)?
5. Let

\[ f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ x + 3 & \text{if } 0 \leq x \end{cases} \quad \text{and} \quad g(x) = |x| - 4 \]

Find a symbolic representation of \( g \circ f(x) \).

6. Solve \(-3 < 2x - 1 < 5\).
7. Find the vertex and sketch the graph of \( y = x^2 + 2x + 4 \).

8. What is the domain of

\[
f(x) = \frac{\sqrt{x^2 - 1}}{x + 4}.
\]
9. Find the center and the radius of the circle given by

\[ x^2 + 6x + y^2 - 4y = -4. \]

10. What is the midpoint of the line segment from \((3, -6)\) to \((5, 12)\)?

11. Suppose \(g\) is defined by \(g(x) = (4 - x)/6\). Let \(f\) be the inverse of the function \(g\). Find a symbolic representation of \(f\).