April 2000

Your name ____________________________

Problems count 5 points each.

1. What is the (implied) domain of the function \( h(x) = (x + 1)^2 \cdot x \cdot \sqrt{4 - x^2} \)?
   Also, how many zeros does it have?

   **Solution:** Domain is \([-2, 2]\) and there are four distinct zeros, \(x = -2, x = -1, x = 0, \) and \(x = 2\).

2. The graph of \( f(x) = ax^2 + bx + c \) is shown for certain values of \(a, b,\) and \(c\). Which of the following quantities are positive? Circle all that apply.

   ![Graph of \( f(x) = ax^2 + bx + c \)]

   (A) \(a\) \hspace{1cm} (B) \(c\) \hspace{1cm} (C) \(b^2 - 4ac\) \hspace{1cm} (D) \(9a + 3b + c\) \hspace{1cm} (E) \(a + b + c\)

   **Solution:** Note that \(c = f(0) = 2\), and \(D = b^2 - 4ac\) is positive because \(f\) has two zeros. Finally, \(f(1) = a + b + c > 2\), so it too is positive.

3. Solve \(\sqrt{5 + x} = 5\sqrt{x}\).

   **Solution:** Square both sides to get \(5 + x = 25x\) or \(24x = 5\), so \(x = 5/24\).

4. Find an equation for the line perpendicular to \(2x - y = 3\) that goes through the point \((1, 3)\).

   **Solution:** The slope is \(-1/2\), so the line is \(y - 3 = -1/2(x - 1)\) or \(y = -x/2 + 7/2\).
5. Solve $x + 1 = 12/x$.

**Solution:** Message the given equation to get $x^2 + x - 12 = 0$ which factors to give the two roots $x = 3$ and $x = -4$.

6. Find an $x$ such that $\log_{10}(6x^2 - 3) = 2$.

**Solution:** Translate this to exponential notation to get $10^2 = 6x^2 - 3$ and solve to get $x = \sqrt{103/6} \approx \pm 4.14$.

7. Solve the inequality $x^3 - x^2 \leq 12x - 2x^2$.

**Solution:** The inequality is equivalent to

$$x^3 + x^2 - 12x \leq 0,$$

the left side of which factors nicely. Use the test-interval idea to get critical points $-4, 0$ and $3$. Then test to find that the solution is $(-\infty, -4] \cup [0, 3]$.

8. What is the time required for a 6% investment compounded monthly to double? ....to triple?

**Solution:** Recall that $A = P(1 + r/n)^{nt}$ where $A$ is the amount, $P$ the principle, $r$ is the annual rate, $t$ is the time in years, and $n$ is the number of compoundings per year. Doubling means $A = 2P$ so we need to solve

$$2 = (1 + 0.06/12)^{12t}$$

or $2 = 1.005^{12t}$. Take logs of both sides to get $t = \ln 2/\ln 1.005 \approx 11.58$ years. The triple time is the solution to $3 = 1.005^{12t}$, so it takes $t = 18.36$ years for such an investment to triple.

9. Solve $\frac{1}{x-1} + \frac{x}{x+1} = 4$.

**Solution:** Find a common denominator and add the fractions to get the quadratic $3x^2 - 5 = 0$ which then leads to $x = \pm \sqrt{5/3} \approx \pm 1.29$.

10. Find

$$\log_4 \left( \frac{8x^3}{2y} \right)$$

in terms of $x$ and $y$.

**Solution:** $\log_4 \left( \frac{8x^3}{2y} \right) = \log_4 8 + 3\log_4 x - \log_4 2 - \log_4 y = \log_4 4 + \log_4 2 - \log_4 2 + 3\log_4 x - \log_4 y = 1 + 3\log_4 x - \log_4 y$. 

2