November 13, 2000

The first 10 problems count 6 points each and rest of the problems count as marked. Problems 1 through 10 are multiple choice and 11 through 15 are fill-in-the-blank. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 2 through 15. There are 122 points available on this test.

1. Which of the following numbers belong to the domain of the function $h(x) = \sqrt{12 + x - x^2}$? Circle all those that apply.
   (A) $-4$  (B) $-3$  (C) $2$  (D) $5$  (E) $7$

2. A line $L$ is parallel to the line whose equation is $2x + 5y = 6$ and passes through the point $(4, -7)$. What is the slope of $L$?
   (A) $-2.5$  (B) $-1$  (C) $-0.4$  (D) $2$  (E) $5$

3. Let $a$ and $b$ be numbers. What is the midpoint of the line segment joining the points $(3a + b, 2b)$ and $(a - b, 2a - 4b)$?
   (A) $(2a, b)$  (B) $(b, a)$  (C) $(2a, a - b)$  (D) $(b, a - b)$  (E) none of A, B, C or D

4. Into how many regions does the solution set $S$ of
   $$(y + x^2)(x^2 + y^2 - 1)(y - x^2) = 0$$
   divide the plane? Note that some of the regions are bounded (surrounded by points of $S$), and some are unbounded.
   (A) $4$  (B) $8$  (C) $10$  (D) $12$  (E) $16$

5. The equation $x^2 - 4x + 2 = 0$ has two solutions. The sum of these two solutions is
   (A) $4 - 2\sqrt{2}$  (B) $2$  (C) $4$  (D) $4\sqrt{2}$  (E) $4 + 2\sqrt{2}$

6. Lindsey went on a long hike. She walked 4 miles north, then 3 miles east, then 2 miles north, and finally 5 miles west. How far was she from her starting point? (Assume the earth is flat, and her hike takes place in North Carolina.)
   (A) $\sqrt{10}$  (B) $\sqrt{17}$  (C) $3\sqrt{3}$  (D) $5$  (E) $2\sqrt{10}$
7. \(|6\sqrt{2} - 9| + |12 - 9\sqrt{2}| =

(A) \(3\sqrt{2} - 3\)  (B) \(3 - 3\sqrt{2}\)  (C) \(21 + 15\sqrt{2}\)  (D) \(3\sqrt{2} + 21\)  (E) none of A, B, C or D

8. What is the product of the roots of \((x - 4)(x - 2) + (x - 4)(x - 5) = 0\)?

(A) 6  (B) 10  (C) 14  (D) 28  (E) 40

9. What is the slope of the secant line joining the points \((-1, f(-1))\) and \((3, f(3))\) for the function defined by \(f(x) = |7 - 2x^2|\)?

(A) 1.5  (B) 2  (C) 3  (D) 4  (E) none of A, B, C or D

10. The circle whose equation is \(x^2 + y^2 - 6x - 4y = 23\) has a radius of

(A) 4  (B) 9/2  (C) 5  (D) 6  (E) 13/2

11. (8 points) What is the vertex of the parabola \(y = 2x^2 - 5x + 8\)?

12. (8 points) Suppose the curve \(y = 3x^2 + bx + 3\) has exactly one \(x\) intercept. Find a value of \(b\)?
13. (12 points) What are the coordinates of the image of the point \((-6, -2)\) when it is reflected

(a) about the \(x\)-axis
(b) about the \(y\)-axis
(c) about the origin
(d) about the line \(y = x\)

14. (14 points) Problems 14 and 15 refer to the functions \(f\) and \(g\). Let

\[
 f(x) = \begin{cases} 
 2x - 3 & \text{if } x < 4 \\
 x + 1 & \text{if } x \geq 4 
\end{cases} \quad \text{and} \quad g(x) = \begin{cases} 
 |x| & \text{if } x < 3 \\
 x^2 - 1 & \text{if } x \geq 3 
\end{cases}
\]

Complete the table of values for \(g\) and \(f \circ g\).

<table>
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<tr>
<th>(x)</th>
<th>(g(x))</th>
<th>(f \circ g(x))</th>
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15. (8 points) Compute \(f((g/f)(3)) + g((f \cdot g)(-1))\).
(12 points) Do exactly one of the next two problems.

16. Find all $x$ satisfying the inequality $\frac{(x-1)(x+3)^2}{(x-2)} \geq 0$.

17. Explain how you can describe the graph of the quadratic equation

$$y = ax^2 + bx + c$$

based on the coefficients $a, b,$ and $c$. Hint: it may be useful to define the discriminant $D$ to be $b^2 - 4ac$. In particular, address the questions (a) does the curve open upwards or downwards, and (b) does it have $x$-intercepts?

18. Bonus Problem (10 pts). If you read my essay Philosophy of Teaching, part 1 at the website, either write a sentence about why I think it is important to read the text or name the doctor referred to in the essay, and say why he is referenced.