1. Suppose the functions $f$ and $g$ are differentiable and their values at certain points are given in the table. The next four problems refer to these functions $f$ and $g$. Notice that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. Note also that, for example, if $K(x) = f(x) - g(x)$, then $K'(x) = f'(x) - g'(x)$ and $K'(4) = f'(4) - g'(4) = 5 - 10 = -5$. Answer each of the questions below about functions that can be build using $f$ and $g$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
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<td>5</td>
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<td>1</td>
<td>7</td>
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<td>4</td>
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<td>4</td>
<td>1</td>
<td>7</td>
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</tr>
</tbody>
</table>

(a) The function $h$ is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(3)$. 
\[ h'(x) = f'(g(x)) \cdot g'(x), \text{ so } h'(3) = f'(g(3)) \cdot g'(3) = f'(2) \cdot 6 = 24. \]

(b) The function $k$ is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(1)$. 
\[ k'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x), \text{ so } k'(1) = 3 \cdot 7 + 3 \cdot 2 = 27. \]

(c) The function $H$ is defined by $H(x) = f(f(x))$. Use the chain rule to find $H'(2)$. 
\[ H'(x) = f'(f(x)) \cdot g'(x), \text{ so } H'(2) = f'(f(2)) \cdot f'(2) = f'(5) \cdot 4 = 16. \]

(d) Let $Q(x) = f(f(x) - g(x))$. Find $Q'(5)$. 
\[ Q'(x) = f'(f(x) - g(x)) \cdot (f'(x) - g'(x)), \text{ so } Q'(5) = f'(6 - 3) \cdot (4 - 3) = f'(3) = 2. \]

(e) Find the derivative of the function $f/g$ at the point $x = 4$. 
\[ \frac{f'(4)g(4) - g'(4)f(4)}{(g(4))^2} = \frac{30 - 30}{36} = 0. \]
2. Suppose that the derivative of the function \( f \) is given by

\[ f'(x) = x^2 - 6x + 5. \]

Note: you are given the derivative function! Answer the following questions about \( f \).

(a) Find an interval over which \( f \) is increasing.

\[ f'(x) = (x - 1)(x - 5) \] so \( f \) is montonic on \((-\infty, 1), (1, 5), \) and \((5, \infty)\).

Observe that \( f' \) is positive over the first and last of these.

(b) Find the location of a relative maximum of \( f \).

\[ f''(1) = -6 < 0 \] implies that \( f \) has a relative max at 1.

(c) Find the location of a relative minimum of \( f \).

\[ f''(5) = 4 > 0 \] implies that \( f \) has a relative min at 5.

(d) Find an interval over which \( f \) is concave upwards.

\[ f''(x) > 0 \] for all \( x > 3 \) implies that \( f \) is concave up on \((3, \infty)\).

(e) Suppose \( f(1) = 3 \). Find \( f(2) \).

\[ f(x) = x^3/3 - 3x^2 + 5x + c \] for some constant \( c \). Solve \( f(1) = 3 \) for \( c \) to get \( c = 2/3 \). Then \( f(2) = 4/3 \).

3. Compute each of the following derivatives.

(a) \[ \frac{d}{dx} \sqrt{x^3 + 1} = \frac{3x^2}{2\sqrt{x^3 + 1}} \]

(b) \[ \frac{d}{dx} \ln(x^3 + 1) = \frac{3x^2}{x^3 + 1} \]

(c) Let \( f(x) = \frac{d}{dx} e^{x^2+1} \cdot e^{2x} \). Find \( f'(x) \).

\[ f'(x) = 2(x + 1)e^{(x+1)^2} \]

(d) \[ \frac{e^x}{x} = \frac{x-1}{x^2}e^x \]

4. Compute the following antiderivatives.
   
   (a) \[ \int 6x^3 - 5x - 1 \, dx = \frac{3}{2} \cdot x^4 - \frac{5}{2} \cdot x^2 - x + c \]

   (b) \[ \int 6x^{\frac{3}{2}} + x^{-\frac{1}{2}} \, dx = 6 \cdot \frac{2}{5} \cdot x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + c \]

   (c) \[ \int \frac{3x^3 + 2x - 1}{x} \, dx = \int 3x + 2 - 1/x \, dx = 3x^2/2 + 2x - \ln |x| + c \]

   (d) \[ \int \frac{2x + 1}{x^2 + x - 3} \, dx = \ln |x^2 + x - 3| + c \]

5. Compute the following definite integrals.
   
   (a) \[ \int_0^2 2xe^{-x^2} \, dx = -e^{-x^2}\bigg|_0^2 = 1 - e^{-4} \approx 0.9816 \]

   (b) \[ \int_0^5 (2x-1)\sqrt{x^2 - x + 5} \, dx = 2/3(x^2 - x + 5)^{3/2}\bigg|_0^5 = \frac{10}{3} (25 - \sqrt{5}) \approx 75.8798 \]

6. Find the largest interval over which \( f(x) = 4x^3 + 39x^2 - 42x \) is decreasing.
   
   \( f'(x) = 12x^2 + 78x - 42 \), so the critical points are \( x = -7 \) and \( x = 1/2 \). Use the test interval method to find that \( f'(x) < 0 \) on the interval \((-7, 1/2)\), so \( f \) is decreasing over that interval.
7. Find a function $G(x)$ whose derivative is $3x^2 - 7$ and for which $G(4) = 9$.

8. Find the area of the region bounded by $y = x^{3/2}$, the $x$-axis, and the lines $x = 0$ and $x = 4$. 
9. Find the area of the region caught between the graphs of the functions

\[ f(x) = -x^2 + 4x \quad \text{and} \quad g(x) = -2x + 5. \]

10. An apartment complex has 100 two-bedroom units for rent all at the same price. The monthly profit from renting \( x \) units is given by

\[ P(x) = -10x^2 + 1760x - 50000 \]

dollars. Find the number of units that should be rented out to maximize the profit. What is the maximum monthly profit realizable?