The multiple choice problems count five points each.

1. Let \( f(x) = 2x^3 - 3x + 8 \). What is \( f'(1) \)?
   \[
   \begin{array}{lllll}
   (A) & 0 & (B) & 3 & (C) & 8 \\
   (D) & 12 & (E) & 21
   \end{array}
   \]

2. Let \( h(x) = e^{x^2} \). What is \( h''(0) \)?
   \[
   \begin{array}{lllll}
   (A) & 0 & (B) & 1 & (C) & 2 \\
   (D) & 4 & (E) & 6
   \end{array}
   \]

3. Let \( g(x) = \ln(1 + \frac{1}{x}) \). What is the slope of the line tangent to the graph of \( g \) at the point \((2, \ln 1.5)\)?
   \[
   \begin{array}{lllll}
   (A) & -1/3 & (B) & -1/6 & (C) & -1/12 \\
   (D) & 1/12 & (E) & 2/3
   \end{array}
   \]

4. The distance from the point \((3, 4)\) to the point \((-1, x)\) is 5. Which of the following could be \( x \)?
   \[
   \begin{array}{lllll}
   (A) & 2 & (B) & 4 & (C) & 5 \\
   (D) & 7 & (E) & 8
   \end{array}
   \]

5. What is \( \lim_{h \to 0} \frac{\sqrt{1+2h}-1}{h} \)?
   \[
   \begin{array}{lllll}
   (A) & -1 & (B) & 0 & (C) & 1/2 \\
   (D) & 1 & (E) & 2
   \end{array}
   \]

6. The slope of the line that contains the points \((-1, y)\) and \((4, -12)\) is \(-2\). What is \( y \)?
   \[
   \begin{array}{lllll}
   (A) & -3 & (B) & -2 & (C) & 3 \\
   (D) & 5 & (E) & 6.2
   \end{array}
   \]

7. At which of the following points is the second derivative of \( x^4 - 6x^3 + 12x^2 + 2x + 2 \) negative?
   \[
   \begin{array}{lllll}
   (A) & -1/2 & (B) & 1/2 & (C) & 3/2 \\
   (D) & 5/2 & (E) & 7/2
   \end{array}
   \]
8. What is the slope of the line perpendicular to the line $2y + x = 6$?

(A) $-3$  (B) $-2$  (C) $-1/2$  (D) $2$  (E) $3$

9. The function $f$ has second derivative given by $f''(x) = 2x - 1$, and also satisfies $f(0) = 19/6$ and $f'(0) = 1$. What is $f(1)$?

(A) $1$  (B) $2$  (C) $3$  (D) $4$  (E) $5$

10. Suppose $f'(x) = 2x^2$ and $g(x) = 3x - 1$. What is $\frac{d}{dx}(f \circ g(x))$?

(A) $2(3x - 1)^2$  (B) $3(9x^2 - 6x + 1)$  (C) $6(9x^2 - 6x + 1)$

(D) $2x^2(3x - 1)$  (E) $4x(3x - 1)$

11. Let

$$f(x) = \begin{cases} 
2 + \sqrt{1-x} & \text{if } x \leq 1 \\
1/(1-x) & \text{if } x > 1 
\end{cases}$$

and let $g(x) = 2x - 1$. Compute $g(f(2) - f(1))$.

(A) $-7$  (B) $-5$  (C) $1$  (D) $1$  (E) $5$

12. What is $\lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h}$?

(A) $-2/3$  (B) $-2/9$  (C) $2/9$  (D) $2/3$  (E) $3/2$

13. What is the number of vertical asymptotes of the function $h$ defined by

$$h(x) = \frac{(x^2 - 1)(x^2 - 4)}{(x-3)(x-2)(x-1)(x+1)(x+2)^2}$$?

(A) $2$  (B) $3$  (C) $4$  (D) $5$  (E) $6$
14. It takes exactly 12 years for $P$ invested at an annual rate $r$ compounded continuously to triple. What is $r$ (to the nearest 0.001)?

(A) 0.075  (B) 0.080  (C) 0.086  (D) 0.092  (E) 0.102

15. What is $(2x - 3) \cdot (x - 1) - (2x - 3) \cdot x - 1$?

(A) 0  (B) 2 - 2x  (C) 2x - 4  (D) 2x - 3  (E) 2x - 2

16. The number $x$ satisfies $2^x = 5$. What is $7^x$?

(A) 90.19  (B) 90.83  (C) 91.09  (D) 91.55  (E) 91.68

17. Suppose $f$ is a continuous function such that $f(0) = -1, f(1) = 2, f(2) = -3, f(3) = 4, f(4) = -2$, and $f(5) = -3$. What is the fewest number of zeros $f$ could have?

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

18. Suppose the function

$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 2 \\ kx - 6 & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$. Then $k =$

(A) 1  (B) 2  (C) 5  (D) 6  (E) 7

19. It takes 10 years for a $1000$ invested at an annual rate of $r$ compounded quarterly to double. What is $r$?

(A) 0.070  (B) 0.072  (C) 0.074  (D) 0.076  (E) 0.078

20. What is $\int_0^3 x^2 + 2x + 1 \, dx$?

(A) 13  (B) 17  (C) 21  (D) 25  (E) 27
21. (20 points) Compute each of the following derivatives.

(a) \( \frac{d}{dx} \sqrt{x^3 + 1} \)

(b) \( \frac{d}{dx} \ln(2x^3 + 1) \)

(c) Let \( f(x) = e^{x^2} \cdot e^{-2x+1} \). Find \( f'(x) \).

(d) \( \frac{d}{dx} \frac{e^{2x}}{x} \)
22. (20 points) Compute the following antiderivatives.

(a) \[ \int \frac{3x^2}{2\sqrt{x^3 + 1}} \, dx \]
(b) \[ \int \frac{x^3 - 2x - 1}{x} \, dx \]
(c) \[ \int \frac{3x^2 + 1}{x^3 + x - 3} \, dx \]

Hint: Let \( u = x^3 + x - 3 \).

23. (16 points) Compute the following integrals.

(a) \[ \int_0^2 2xe^{-x^2} \, dx \]
(b) \[ \int_0^5 (2x - 1)\sqrt{x^2 - x + 5} \, dx \]

24. (10 points) Find a function \( G(x) \) whose derivative is \( 1/(x - 5) \) and whose value at \( x = 6 \) is 9.

25. (10 points) Find the area of the region bounded by \( y = x^{3/2} \), the \( x \)-axis, and the lines \( x = 0 \) and \( x = 4 \).

26. (20 points) A 16in. by 12in. sheet of paper is used to build a topless box as follows: an \( x \)-in. by \( x \)-in. square is cut from each corner, and the resulting rectangular pieces are folded upward along the dotted lines to form the sides of the box.

(a) What is the volume \( V \) of the resulting box?
(b) Find \( \frac{d}{dx}V(x) \).
(c) What is the domain of \( V \)?
(d) Find all stationary points of \( V \).
(e) What value of \( x \) maximizes the volume?
(f) What is the maximal volume?
27. (20 points) According to Newton's Law of Cooling, the temperature \( F(t) \) of a body in a surrounding medium changes at a rate that is proportional to the difference between the temperature of the body and the temperature of the surroundings. It follows that \( F(t) = T + Ae^{-kt} \), where \( t \) is expressed in minutes, \( T \) is the temperature in Celsius of the surrounding medium, and \( A \) and \( k \) are constants. A hard-boiled egg at 98°C is put in a pan under running 10°C water to cool. After 5 minutes, the egg’s temperature is found to be 38°C. How much longer will it take the egg to reach 20°C? Use the following steps to solve the problem. Show your work in detail.

(a) What is \( T \)?

(b) Use the fact that \( f(0) = 98^\circ \) and the value of \( T \) to find \( A \).

(c) Use the values of \( T \) and \( A \) and the temperature of the egg after five minutes to find the value of \( k \).

(d) Use the values of \( A, T, \) and \( k \) to find the time required for the egg to become 20°C.

Solutions and answers.

1. Let \( f(x) = 2x^3 - 3x + 8 \). What is \( f'(1) \)?
   \[
   \begin{array}{llllll}
   & (A) & 0 & (B) & 3 & (C) & 8 & (D) & 12 & (E) & 21 \\
   \end{array}
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2. Let \( h(x) = e^{x^2} \). What is \( h''(0) \)?
   \[
   \begin{array}{llllll}
   & (A) & 0 & (B) & 1 & (C) & 2 & (D) & 4 & (E) & 6 \\
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   \begin{array}{llllll}
   & (A) & 2 & (B) & 4 & (C) & 5 & (D) & 7 & (E) & 8 \\
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5. What is \( \lim_{h \to 0} \frac{\sqrt{1 + 2h} - 1}{h} \)?
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   \begin{array}{llllll}
   & (A) & -1 & (B) & 0 & (C) & 1/2 & (D) & 1 & (E) & 2 \\
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6. The slope of the line that contains the points \((-1, y)\) and \((4, -12)\) is \(-2\). What is \(y\)?

(A) \(-3\)  (B) \(-2\)  (C) 3  (D) 5  (E) 6.2

7. At which of the following points is the second derivative of 
\[x^4 - 6x^3 + 12x^2 + 2x + 2\]

negative?

(A) \(-1/2\)  (B) 1/2  (C) \(3/2\)  (D) 5/2  (E) 7/2

8. What is the slope of the line perpendicular to the line \(2y + x = 6\)?

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(A) \(-7\)  (B) \(-5\)  (C) \(-1\)  (D) 1  (E) 5

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(A) \(-2/3\)  (B) \(-2/9\)  (C) 2/9  (D) 2/3  (E) 3/2

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(c) Let $f(x) = e^{x^2} \cdot e^{-2x + 1}$. Find $f'(x)$.  $2(x - 1)e^{x-1}^2$
(d) \( \frac{d}{dx} \frac{e^{2x}}{x} = \frac{(2x-1)e^{2x}}{x^2} \)

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