As usual, show all your work. If you used a calculator, explain in detail how you used it to solve the problem. There are 237 points available on this test.

1. (42 points) Find the following antiderivatives.

   (a) $\int 2x - 3 \, dx$

   (b) $\int 6x^2 - 4x - 1 \, dx$

   (c) $\int \frac{x^3 + 2x - 1}{x} \, dx$

   (d) $\int \frac{4x + 1}{2x^2 + x - 3} \, dx$

   (e) $\int 5x^4(x^5 + 3)^7 \, dx$

   (f) $\int 3x^2 e^{x^3} \, dx$
2. (30 points) Note that \( g(x) = (x - 1)(x - 3) \) has two zeros in the interval \([0, 4]\).

(a) Find the area of the ‘triangular’ region bounded by (i) the \(x\)-axis, (ii) the line \(x = 4\), and (iii) the graph of \(g(x)\).

(b) Compute \( \int_0^4 g(x) \, dx \).

(c) Find the area of the region caught between the graph of \(g(x)\) and the \(x\)-axis over the interval from \(x = 0\) to \(x = 4\). Explain why this is different from the number found in part b.

3. (20 points) Find a function \(G(x)\) whose derivative is \(3x^2 - 7x + 3\) and for which \(G(2) = -3\).
4. (40 points) Let \( f(x) = \sqrt{x^2 + 1}, g(x) = \frac{x+1}{x-1}, \) and \( h(x) = 2x - 3. \) Find each of the functions.

(a) \( \frac{d}{dx} (f \circ g(x)) \)

(b) \( h'(g'(x)) \)

(c) \( \frac{d}{dx} (h \circ h(x)) \)

(d) \( \frac{d}{dx} (h(x) \cdot (g(x))^2) \)

(e) \( \frac{d}{dx} (h(x) \div g(x)) \)
5. (20 points) Let \( g(x) \) be defined as follows: Let
\[
g(x) = \begin{cases} 
  e^{2x} & \text{if } x \leq 1 \\
  \ln(x - 1) & \text{if } x > 1
\end{cases}
\]

(a) Compute the derivative of \( g(x) \).

(b) What is the slope of the line tangent to the graph of \( g(x) \) at the point \((0, 1)\).

(c) What is the slope of the line tangent to the graph of \( g(x) \) at the point \((3, \ln(2))\).

(d) Find an equation for the line tangent to the graph of \( g(x) \) at the point \((3, \ln(2))\).
6. (40 points) Suppose \( u(x) \) is a function whose derivative is

\[
u'(x) = (x^2 - 4)(x - 1)^2(x + 3)(x + 5).
\]

Recall that a major theorem tells you the intervals over which \( u(x) \) is increasing based on \( u'(x) \).

(a) Find the critical points of \( u(x) \).

(b) Use the Test Interval Technique to find the intervals over which \( u(x) \) is increasing.
7. (25 points) Consider the function \( h(x) = \sqrt{2x^3 - 3x^2 - 36x + 500} \) defined over the interval \([-5, 5]\).

(a) Find \( h'(x) \).

(b) Find the critical points of \( h(x) \).

(c) Find the absolute maximum and absolute minimum of the \( h(x) \) over its domain.

8. (20 points) Compute the following definite integrals.

(a) \( \int_{0}^{2} 2xe^{-x^2} \, dx \)

(b) \( \int_{0}^{5} (2x - 1)\sqrt{x^2 - x + 5} \, dx \)