As usual, show all your work. If you used a calculator, explain in detail how you used it to solve the problem. There are 237 points available on this test.

1. (42 points) Find the following antiderivatives.

   (a) \( \int (2x - 3) \, dx \)
   
   Solution: \( x^2 - 3x + C \).

   (b) \( \int 6x^2 - 4x - 1 \, dx \)
   
   Solution: \( 2x^3 - 2x^2 - x + C \).

   (c) \( \int \frac{x^3 + 2x - 1}{x} \, dx \)
   
   Solution: \( \int \frac{x^2 + 2 - 1}{x} \, dx = \frac{x^3}{3} + 2x - \ln |x| + C \).

   (d) \( \int \frac{4x + 1}{2x^2 + x - 3} \, dx \)
   
   Solution: By substitution, \( u = 2x^2 + x - 3 \), \( f \frac{4x+1}{2u^2+u-3} \, dx = \ln |2x^2 + x - 3| + C \).

   (e) \( \int 5x^4 (x^5 + 3)^7 \, dx \)
   
   Solution: By substitution with \( u = x^5 + 3 \), \( f 5x^4 (x^5 + 3)^7 \, dx = \frac{(x^5+3)^8}{8} + C \).

   (f) \( \int 3x^2 e^{x^3} \, dx \)
   
   Solution: By substitution with \( u = x^3 \), \( du = 3x^2 \), \( f e^u \, du = e^u + C = e^{x^3} + C \).
2. (30 points) Note that \( g(x) = (x - 1)(x - 3) \) has two zeros in the interval \([0, 4]\).

(a) Find the area of the ‘triangular’ region bounded by (i) the \(x\)-axis, (ii) the \(y\)-axis, and (iii) the graph of \( g(x) \).

Solution:

(b) Compute \( \int_{0}^{4} g(x) \, dx \).

Solution:

(c) Find the area of the region caught between the graph of \( g(x) \) and the \(x\)-axis over the interval from \( x = 0 \) to \( x = 4 \). Explain why this is different from the number found in part b.

Solution:

3. (20 points) Find a function \( G(x) \) whose derivative is \( 3x^2 - 7x + 3 \) and for which \( G(2) = -3 \).

Solution: \( G(x) = x^3 - 7x^2/2 + 3x + C \) for some constant \( C \). But since \( G(2) = -3 = 2^3 - 28/2 + 6 + C \), it follows that \( C = 8 - 14 + 6 = 0 \) and \( G(x) = x^3 - 7x^2/2 + 3x \).
4. (40 points) Let \( f(x) = \sqrt{x^2 + 1}, \) \( g(x) = \frac{x+1}{x-1}, \) and \( h(x) = 2x - 3. \) Find each of the functions.

(a) \( \frac{d}{dx} (f \circ g(x)) \)

\[ \text{Solution:} \] By the chain rule,

(b) \( h'(g'(x)) \)

\[ \text{Solution:} \]

(c) \( \frac{d}{dx} (h \circ h(x)) \)

\[ \text{Solution:} \]

(d) \( \frac{d}{dx} (h(x) \cdot (g(x))^2) \)

\[ \text{Solution:} \]

(e) \( \frac{d}{dx} (h(x) \div g(x)) \)

\[ \text{Solution:} \]
5. (20 points) Let \( g(x) \) be defined as follows: Let

\[
  g(x) = \begin{cases} 
    e^{2x} & \text{if } x \leq 1 \\
    \ln(x - 1) & \text{if } x > 1 
  \end{cases}
\]

(a) Compute the derivative of \( g(x) \).

Solution:

(b) What is the slope of the line tangent to the graph of \( g(x) \) at the point \((0, 1)\).

Solution:

(c) What is the slope of the line tangent to the graph of \( g(x) \) at the point \((3, \ln(2))\).

Solution:

(d) Find an equation for the line tangent to the graph of \( g(x) \) at the point \((3, \ln(2))\).

Solution: First note that \( g'(3) = 1/2 \) and that \( g(3) = \ln(2) \) so the line is \( y - \ln(2) = \frac{1}{2}(x - 3) \), which simplifies to \( y = \frac{x - 3}{2} + \ln(2) = \frac{x}{2} - \frac{3}{2} + \ln(2) \).
6. (40 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 4)(x - 1)^2(x + 3)(x + 5).$$

Recall that a major theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.

(a) Find the critical points of $u(x)$.

**Solution:** $x = -2, x = 2, x = 1, x = -3$ and $x = -5$.

(b) Use the Test Interval Technique to find the intervals over which $u(x)$ is increasing.

**Solution:** $u$ is increasing over $(-\infty, -5), (-3, -2)$ and over $(2, \infty)$. 
7. (25 points) Consider the function \( h(x) = \sqrt{2x^3 - 3x^2 - 36x + 500} \) defined over the interval \([-5, 5]\).

(a) Find \( h'(x) \).

**Solution:** Differentiate to get \( h'(x) = 6(x - 3)(x + 2)(2x^3 - 3x^2 - 36x + 500)^{-1/2} \)

(b) Find the critical points of \( h(x) \).

**Solution:** The only critical points are \( x = 3 \) and \( x = -2 \).

(c) Find the absolute maximum and absolute minimum of the \( h(x) \) over its domain.

**Solution:** Checking endpoints, we have \( h(-5) = \sqrt{355} \) and \( h(5) = \sqrt{495} \). Also, \( h(3) = \sqrt{419} \) and \( h(-2) = \sqrt{544} \). So the minimum value is \( \sqrt{355} \) which occurs at \( x = -5 \) and the maximum value is \( \sqrt{544} \) which occurs at \( x = -2 \).

8. (20 points) Compute the following definite integrals.

(a) \( \int_0^2 2xe^{-x^2} \, dx \)

**Solution:** \(-e^{-x^2}]_0^2 = 1 - e^{-4} \approx 0.9816.\)

(b) \( \int_0^5 (2x - 1)\sqrt{x^2 - x + 5} \, dx \)

**Solution:** \( 2/3(x^2 - x + 5)^{3/2}]_0^5 = \frac{10}{3}(25 - \sqrt{5}) \approx 75.8798.\)