Find a function $f$ satisfying the following two conditions:

a. $f'(x) = 4x^2 - 7x + 5$, and  
b. $f(6) = 0$.

Solution. The antiderivative of $f'(x)$ is obtained as follows:

$$\int 4x^2 - 7x + 5 \, dx = \int 4x^2 \, dx - \int 7x \, dx + \int 5 \, dx = 4 \int x^2 \, dx - 7 \int x \, dx + 5 \int 1 \, dx = 4 \cdot \frac{x^3}{3} - 7 \cdot \frac{x^2}{2} + 5x + C.$$  

Thus $C$ must satisfy the equation

$$0 = f(6) = 4 \cdot \frac{6^3}{3} - 7 \cdot \frac{6^2}{2} + 5 \cdot 6 + C.$$  

Solving this for $C$ yields $C = -192$. Thus $f(x) = 4 \cdot \frac{x^3}{3} - \frac{7x^2}{2} + 5x - 192$. 
