Let $f(x) = x^2 + 1$. Compute the derivative $f'$ of $f$ using the definition of derivative.

Solution: We need to find
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.
\]
So first find the difference quotient, $\frac{f(x + h) - f(x)}{h}$.

\[
\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 + 1 - (x^2 + 1)}{h}
\]
\[
= \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}
\]
\[
= \frac{2xh + h^2}{h}
\]
\[
= \frac{h(2x + h)}{h}
\]
\[
= 2x + h
\]

Next, take the limit as $h$ approaches 0 to get $2x$. 

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