1. Which of the following numbers belong to the (implied) domain of
\[ f(x) = \frac{\sqrt{x-2}}{x-3}? \]
Circle all those that apply.

(A) -2 (B) 2 (C) 3 (D) 4 (E) 5

Solution: The domain includes all real numbers greater than or equal to 2 except 3, which makes the denominator zero. Thus, 2, 4, and 5 all belong to the domain.

2. What is the \( y \)-intercept of the line defined by \( \frac{x}{6} + \frac{y}{3} = 2 \)?

(A) -2 (B) 4 (C) 6 (D) 12 (E) 16

Solution: The \( y \)-intercept is the point on the line for which \( x = 0 \). Solving for \( y \) gives \( y = 6 \).

3. Let \( f(x) = 2x + 4 \) and \( g(x) = 3x - 9 \). What is the value of \( g(f(g(3))) \)?

(A) -18 (B) -3 (C) 3 (D) 9 (E) 18

Solution: \( g(f(g(3))) = g(f(0)) = g(4) = 3 \).

4. Let \( f(x) = x^2 + 1 \). Evaluate and simplify \( \frac{f(x+h)-f(x)}{h} \).

(A) \( h - 2 \) (B) \( 2x - 2h + h^2 \) (C) \( 2x + h \)
(D) \( 2x + h + 2 \) (E) \( x^2 + 2h + 2 \)

Solution: Simplify \( \frac{(x+h)^2+1-(x^2+1)}{h} \) to get \( \frac{x^2+2xh+h^2+1-x^2-1}{h} = \frac{2xh+h^2}{h} \), whereupon, the \( h \) can be factored from the numerator and cancelled with the \( h \) in the denominator to yield \( 2x + h \).

5. Referring to the function \( h(x) \) defined in problem 9, what is the slope of the secant line joining the points \((-2, h(-2))\) and \((4, h(4))\)?

(A) -1 (B) -1/2 (C) 0 (D) 1/2 (E) 1

Solution: A. The slope is \( m = \frac{3-0}{2-4} = -\frac{1}{2} \).
Suppose the functions $f$ and $g$ are given completely by the table of values shown.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>5</td>
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<tr>
<td>1</td>
<td>7</td>
<td>1</td>
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<td>7</td>
<td>4</td>
<td>7</td>
<td>0</td>
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</tbody>
</table>

6. Solve the equation $f \circ g(x) = 6$?

(A) 0   (B) 1   (C) 4   (D) 5   (E) 6

**Solution:** Since $f(5) = 6$, it must be the case that $g(x) = 5$. This is true only when $x = 0$.

7. Compute $(g \circ f)(2 + f(2))$?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

**Solution:** Note that $2 + f(2) = 7$ and $g(f(7)) = g(4) = 6$.

On all the following questions, show your work.

8. (10 points) The supply and demand curves are given below for digital cameras at XYZ Distributors, where $x$ represents the number of units and $p$ the price. Find the equilibrium quantity and price. Demand: $p = -x^2 - 2x + 100$ and Supply: $p = 8x + 25$.

**Solution:** Solve $-x^2 - 2x + 100 = 8x + 25$ by solving the quadratic $-x^2 - 10x + 75 = 0$ to get two solutions, $x = 5$ and $x = -15$, the later of which is extraneous. Thus $x = 5$ and $p = 65$. 

2
9. (30 points) For each of the next questions, let \( h \) be defined as follows:

\[
h(x) = \begin{cases} 
  x^2 - 1 & \text{if } x < 0 \\
  x & \text{if } 0 \leq x < 2 \\
  3 & \text{if } x = 2 \\
  4 - x & \text{if } 2 < x 
\end{cases}
\]

(a) What is \( \lim_{x \to -1} h(x) \)?

\[ \text{Solution: } \lim_{x \to -1} h(x) = (-1)^2 - 1 = 0 \]

(b) What is \( \lim_{x \to 0^-} h(x) \)?

\[ \text{Solution: } \lim_{x \to 0^-} h(x) = \lim_{x \to 0^-} x^2 - 1 = -1 \]

(c) What is \( \lim_{x \to 1} h(x) \)?

\[ \text{Solution: } \lim_{x \to 1} h(x) = \lim_{x \to 0^-} x = 1 \]

(d) What is \( \lim_{x \to 2^+} h(x) \)?

\[ \text{Solution: } \lim_{x \to 2^+} h(x) = \lim_{x \to 2^-} 4 - x = 2 \]

(e) What is \( \lim_{x \to 2} h(x) \)?

\[ \text{Solution: Since } \lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} x = 2 \text{ and the limit from the right is also } 2, \text{ it follows that the limit is } 2. \]

(f) What is \( \lim_{x \to 4} h(x) \)?

\[ \text{Solution: } \lim_{x \to 4} h(x) = \lim_{x \to 4} 4 - x = 0 \]
10. (40 points) Compute each of the following limits.

(a) Let \( f(x) = \begin{cases} 
  x + 2 & \text{if } x \neq 1 \\
  1 & \text{if } x = 1 
\end{cases} \)

\[ \lim_{x \to 1} f(x) \]

**Solution:** Use the blotter test to see that \( f(x) \) is close to 3 when \( x \) is close (but not equal) to 1.

(b) \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \)

**Solution:** Factor the numerator and cancel out the factor \( x - 2 \) to get

\[ \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{x + 2}{1} = 4. \]

(c) \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)

**Solution:** Factor the denominator and cancel out the factor \( x - 1 \) to get

\[ \lim_{x \to 1} \frac{1}{x^2 + x + 1} = \frac{1}{3}. \]

(d) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \)

**Solution:** Rationalize the numerator to get

\[ \frac{\sqrt{x} - 3}{x - 9} = \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} \]

which has limit \( 1/6 \) as \( x \) approaches 9.

(e) \( \lim_{x \to 1} \frac{2x}{x - 1} - \frac{1}{x - 1} \)

**Solution:** Do the fraction arithmetic to get

\[ \frac{2x}{x - 1} - \frac{1}{x - 1} = \frac{2x - 1}{x - 1} = \frac{1}{2x} \]

which has limit \(-1/2\) as \( x \) approaches 1.

(f) \( \lim_{x \to 2} \frac{x^2 - 2x}{x^2 + x - 6} \)

**Solution:** Factor and eliminate the common factor \( x - 2 \), then set \( x = 2 \) to get

\[ 2/(2 + 3) = 2/5. \]

(g) \( \lim_{x \to 2} 2x^3 \sqrt{x^2 + 12} \)

**Solution:** Just replace all the \( x \)'s with the number 2 to get

\[ 2 \cdot 2^3 \sqrt{4 + 12} = 16 \cdot 4 = 64. \]

(h) \( \lim_{x \to \infty} \frac{2x^2}{1 + x^2} \)

**Solution:** We are looking for the horizontal asymptote, which by the asymptote theorem is just \( 2/1 = 2. \)