1. Questions (a) through (e) refer to the graph of the function $f$ given below.

(a) $\lim_{x \to 1^-} f(x) =$

(A) 0  (B) 1  (C) 2  (D) 4  (E) does not exist

**Solution:** Use the blotter test by covering up the left part and then the right part to determine the one-sided limits, both of which are 1. Therefore, $\lim_{x \to 1^-} f(x) = 1$.

(b) $\lim_{x \to 2^-} f(x) =$

(A) 0  (B) 1  (C) 2  (D) 4  (E) does not exist

**Solution:** Again use the blotter test by covering up the right part to determine the one-sided limits, both of which are 1. Therefore, $\lim_{x \to 2^-} f(x) = 0$

(c) A good estimate of $f'(0)$ is

(A) $-1$  (B) 0  (C) 1  (D) 2  (E) there is no good estimate

**Solution:** The tangent line seems to be horizontal, therefore the best estimate of its slope is 0.
(d) A good estimate of $f'(-1)$ is

(A) $-1$  (B) $0$  (C) $1$  (D) $2$  (E) there is no good estimate

**Solution:** The tangent line has a negative slope, therefore the best estimate of its slope is $-1$.

(e) A good estimate of $f'(2)$ is

(A) $-1$  (B) $0$  (C) $1$  (D) $2$  (E) there is no good estimate

**Solution:** There is no tangent line, so there is no good estimate of $f'(2)$.

2. The line tangent to the graph of a function $f$ at the point $(2,3)$ on the graph also goes through the point $(-2,7)$. What is $f'(2)$?

(A) $-2$  (B) $-1$  (C) $0$  (D) $1$  (E) $2$

**Solution:** The slope of the tangent line is $\frac{2-(-2)}{3-(-2)} = -1$, so $f'(2) = -1$.

3. What is the slope of the tangent line to the graph of $f(x) = 2x^{-2}$ at the point $(2,1/2)$?

(A) $-1/2$  (B) $-1/4$  (C) $-1/8$  (D) $-1/16$  (E) $-1/512$

**Solution:** $f'(x) = -4x^{-3}$, which at $x = 2$ has the value $-1/2$.

On all the following questions, **show your work**.

4. (15 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If $f$ is a continuous function on the interval $[a,b]$ and $M$ is a number between $f(a)$ and $f(b)$, then there exists a number $c$ satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{2x - 1}$ and let $[a,b] = [1,5]$. Finally, suppose $M = 2$. Find the number $c$ whose existence is guaranteed by IVT.

**Solution:** Solve the equation $\sqrt{2x - 1} = 2$ by squaring both sides. You get $x = 5/2$. 

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5. (15 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing $x$ playing records is given by $C(x) = 2000 + 3x - 0.01x^2$ for $x$ in the range 0 to 6000.

(a) Find the marginal cost function $C'(x)$.

**Solution:** $C'(x) = 3 - 0.02x$

(b) Find the average cost function $\overline{C}(x)$.

**Solution:** $\overline{C}(x) = \frac{2000 + 3x - 0.01x^2}{x} = \frac{2000}{x} + 3 - 0.01x$.

(c) Find the marginal average cost function $\overline{C}'(x)$.

**Solution:** $\overline{C}'(x) = -2000x^{-2} - 0.01$.

(d) Interpret your results in (c). Is the average cost growing or falling as the company produces more units?

**Solution:** The function $\overline{C}'(x) = -2000x^{-2} - 0.01$ is negative throughout its domain. This means that the average cost decreases the more records are produced.

6. (15 points) Let $f(x) = 4/x$.

(a) Construct $\frac{f(3+h) - f(3)}{h}$

**Solution:** Note that $\frac{f(3+h) - f(3)}{h} = \frac{\frac{4}{h} - 4}{h} = \frac{4}{h + 3h - 4} = \frac{12 - 4(3+h)}{3(3+h)}h$.

(b) Simplify and take the limit of the expression in (a) as $h$ approaches 0 to find $f'(3)$.
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Solution: continued from above: \( \frac{12-12-4h}{3(3+h)h} = \frac{-4h}{3(3+h)h} = -\frac{4}{3(3+h)}. \) Therefore, \( \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{-4}{3(3+h)} = -\frac{4}{9}. \)

(c) Use the information found in (b) to find an equation for the line tangent to the graph of \( f \) at the point \( (3, 4/3) \).

Solution: Use the point-slope formula to get \( y - \frac{4}{3} = -\frac{4}{9}(x - 3) \) which in slope-intercept form is \( y = -\frac{4}{9}x + \frac{8}{3} \).

7. (25 points) Compute the following derivatives.

(a) Let \( f(x) = x^3 + x^{-\frac{3}{2}} \). Find \( \frac{d}{dx} f(x) \).

Solution: \( \frac{d}{dx} f(x) = 3x^2 - \frac{3}{2}x^{-\frac{5}{2}}. \)

(b) Let \( g(x) = \sqrt{x^2 + 4} \). What is \( g'(x) \)?

Solution: \( g'(x) = \frac{1}{2}(x^2 + 4)^{-\frac{1}{2}} \cdot 2x. \)

(c) Find \( \frac{d}{dx} ((3x + 1)^2 \cdot (4x^4 - 1)) \)

Solution: \( \frac{d}{dx} ((3x + 1)^2 \cdot (4x^4 - 1)) = \frac{d}{dx} ((3x + 1)^2) \cdot (4x^4 - 1) + (3x + 1)^2 \frac{d}{dx} (4x^4 - 1) = 2(3x + 1)(4x^4 - 1) + (3x + 1)^2 \cdot 16x^3. \)

(d) Find \( \frac{d}{dx} \frac{2x^2 + 1}{x + 2} \)

Solution: Use the quotient rule to get \( \frac{\frac{d}{dx} (4x+1)(x+2) - (2x^2+1)}{(x+2)^2} = \frac{2x^2+9x}{(x+2)^2}. \)

(e) Find \( \frac{d}{dt} (t^2 + 1/t)^2 \).

Solution: \( \frac{d}{dt} (t^2 + 1/t)^2 = 2(t^2 + t^{-1})(2t - t^{-2}) = 2(2t^3 + 1 - t^{-3}). \)