1. (20 points) Use the definition of derivative to find $f'(a)$ for the function $f(x) = 4x - x^3$. Use this information to find the slope of the line tangent to the graph of $f$ at the point $(-1, -3)$.

Solution:

$$
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \\
\lim_{h \to 0} \frac{4(x + h) - (x + h)^3 - (4x - x^3)}{h} = \\
\lim_{h \to 0} \frac{4x + 4h - (x^3 + 3x^2h + 3xh^2 + h^3) - 4x + x^3}{h} = \\
\lim_{h \to 0} \frac{4h - 3x^2h - 3xh^2 - h^3}{h} = \\
\lim_{h \to 0} \frac{h(4 - 3x^2 - 3xh - h^2)}{h} = \\
\lim_{h \to 0} 4 - 3x^2 - 3xh - h^2 = 4 - 3x^2,
$$

so the slope of the tangent line at $(-1, -3)$ is $f'(-1) = 4 - 3(-1)^2 = 1$.

2. (10 points) Find the derivative of $f(x) = (2x^2 - \sqrt{x})^2$.

Solution: By the chain rule, $f'(x) = 2(2x^2 - \sqrt{x})(4x - \frac{1}{2}x^{-1/2})$.

3. (10 points) Find $\frac{dy}{dx}$ when $y = (x^2 - 7x + 1)(3x - 1/x)$

Solution: By the product rule, $\frac{dy}{dx} = (2x - 7)(3x - 1/x) + (3 + x^{-2})(x^2 - 7x + 1)$. 
4. (10 points) Find an equation for the line tangent to the graph of \( h(x) = \frac{3x - 2}{x^2 - 1} \) at the point \((0, 2)\).

**Solution:** By the quotient rule, 
\[
h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2} = \frac{3(x^2 - 1) - 2x(3x - 2)}{(x^2 - 1)^2} = \frac{3x^2 - 3 - 6x^2 + 4x}{(x^2 - 1)^2} = \frac{-3x^2 + 4x - 3}{(x^2 - 1)^2}.
\]
Thus \( h'(0) = -3/1 = -3 \) and the tangent line is given by \( y - 2 = -3(x - 0) \) which simplifies to \( y = -3x + 2 \).

5. (10 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing \( x \) playing records is given by \( C(x) = 3000 + 3x - 0.001x^2, \ 0 \leq x \leq 6000 \).

(a) Find the average cost function \( \overline{C} \).

**Solution:** 
\[
\overline{C} = \frac{C(x)}{x} = \frac{3000 + 3x - 0.001x^2}{x} = 3000x^{-1} + 3 - 0.001x.
\]

(b) Find the marginal average cost function \( C' \).

**Solution:** 
\[
\overline{C}' = -3000x^{-2} - 0.001.
\]

6. (10 points) Does the function \( f(x) = \sqrt{x + 3} \) satisfy the hypothesis of Intermediate Value Theorem over the interval \([-2, 6]\). If so, find an INTEGER (ie, a whole number) \( M \) between \( f(-2) \) and \( f(6) \), and then find a number \( c \) in the interval \((-2, 6)\) such that \( f(c) = M \).

**Solution:** The only integer between \( f(-2) = \sqrt{-2 + 3} = 1 \) and \( f(6) = \sqrt{6 + 3} = 3 \) is 2, so \( M = 2 \). We need to solve \( f(c) = \sqrt{c + 3} = 2 \). Squaring both sides yields \( c + 3 = 4 \), and it follows that \( c = 1 \).
7. (10 points) Suppose that $f'(3) = 2$ and $f(3) = 1$. What is the $y$-intercept of the line tangent to the graph of $f$ at the point $(3,1)$?

**Solution:** $y - 1 = 2(x - 3)$ is equivalent to $y = 2x - 5$ so the $y$-intercept is $-5$.

8. (30 points) Suppose the functions $f$ and $g$ are differentiable. Some of the values of $f, f', g, \text{ and } g'$ are given in the table. The next six problems refer to these functions $f$ and $g$. Recall that, for example, the entry 10 in the fifth row and sixth column means that $g'(4) = 10$.

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<th>$f'(x)$</th>
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</table>

(a) The function $h$ is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(3)$.

**Solution:** Since $h'(x) = f'(g(x)) \cdot g'(x)$ for all $x$, $h'(3) = f'(g(3)) \cdot g'(3) = f'(2)g'(3) = 4 \cdot 6 = 24$.

(b) The function $R$ is defined by $R(x) = g(f(x))$. Use the chain rule to find $R'(2)$.

**Solution:** Since $R'(x) = g'(f(x)) \cdot f'(x)$ for all $x$, $R'(3) = g'(f(2)) \cdot f'(2) = g'(5)f'(2) = 4 \cdot 4 = 16$.

(c) The function $k$ is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(5)$.

**Solution:** Since $k'(x) = f'(x)g(x) + g'(x)f(x)$, $k'(5) = f'(5)g(5) + g'(5)f(5) = 36$.

(d) The function $H$ is defined by $H(x) = f(x)/g(x)$. Use the quotient rule to find $H'(4)$.

**Solution:** Since $H'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$ for all $x$, $H'(4) = -\frac{4 \cdot 3}{36} = -\frac{1}{3}$.

(e) The function $K$ is defined by $K(x) = (f(x) + g(x))^2$. Find $K'(6)$.

**Solution:** By the chain rule, $K'(x) = 2(f(x) + g(x)) \cdot (f'(x) + g'(x))$ for all $x$, $K'(6) = 2(0 + 1)(5 + 2) = 14$.
(f) The function $M$ is defined by $M(x) = f(f(x))$. Use the chain rule to find $M'(0)$.

**Solution:** By the chain rule, $M'(x) = f'(f(x)) \cdot f'(x)$ for all $x$. Thus, $M'(0) = f'(f(0)) \cdot f'(0) = f'(2) \cdot f'(0) = 4 \cdot 1 = 4$.

9. (20 points) The altitude of a rocket $t$ seconds into flight is given

$$s = f(t) = -2t^3 + 114t^2 + 480t + 1 \quad (t \geq 0),$$

where $s$ is measured in feet.

(a) Find an expression $v$ for the rocket’s velocity at any time $t$.

**Solution:** $v(t) = s'(t) = -6t^2 + 228t + 480$.

(b) Compute the rocket’s velocity when $t = 10, 40, 50, \text{ and } 70$. Interpret your results.

**Solution:** $v(10) = 2160$, $v(40) = 0$, $v(50) = -3120$, and $v(70) = -12960$.

(c) Using the results from part b., find the maximum height of the rocket. Hint: At its maximum height, the velocity of the rocket is zero.

**Solution:** Since $v(40) = 0$, it follows that the maximum height is attained at $t = 40$ seconds. The position of the rocket after 40 seconds is $s(40) = -2 \cdot 40^3 + 114 \cdot 40^2 + 480 \cdot 40 + 1 = 73601$ feet.