1. Questions (a) through (e) refer to the graph of the function $f$ given below.

(a) $\lim_{x \to 1} f(x) =$

(A) 0  (B) 1  (C) 2  (D) 4  (E) does not exist

Solution: B. The limit is 1 by the blotter test.

(b) $\lim_{x \to -2^+} f(x) =$

(A) 0  (B) 1  (C) 2  (D) 4  (E) does not exist

Solution: C. Looking just at the part of the graph that lies to the right of 2, we see that the right limit as $x$ approaches 2 is 2.

(c) A good estimate of $f'(-1)$ is

(A) $-1$  (B) 0  (C) 1  (D) 2  (E) there is no good estimate

Solution: A. The slope of the tangent line is certainly negative, and $-1$ is the only negative option.

(d) A good estimate of $f'(-2)$ is

(A) $-1$  (B) 0  (C) 1  (D) 2  (E) there is no good estimate

Solution: B. The slope of the tangent line is close to zero, so 0 is a reasonable estimate.
(e) A good estimate of \( f'(3) \) is

(A) \(-1\)  (B) 0  (C) 1  (D) 2  (E) there is no good estimate

**Solution:** B. The function is horizontal at 3 so the slope of the tangent line is zero.

2. The line tangent to the graph of a function \( f \) at the point \((2, -3)\) on the graph also goes through the point \((-1, 6)\). What is \( f'(2) \)?

(A) \(-3\)  (B) \(-1\)  (C) 0  (D) 1  (E) 3

**Solution:** A. The derivative is the slope of the tangent line, so it is \( (6 - (-3)) / (-1 - 2) = -3 \)

3. True-false questions. These count 2 points each.

(a) True or false. If \( f \) and \( g \) are differentiable and \( a \) and \( b \) are constants, then \( \frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x) \).

**Solution:** True. This is just the rule that talks about the derivative of the sum and of a constant times a function.

(b) True or false. If \( f'(x) > 0 \) for each \( x \) in the interval \((-1, 1)\), then \( f \) is increasing on \((-1, 1)\).

**Solution:** True.

(c) True or false. If \( f(a) < 0, f(b) > 0 \), and \( f(x) \) is continuous for each \( x \) in \([a, b]\), then there is at least one number \( c \) in \((a, b)\) such that \( f(c) = 0 \).

**Solution:** True. The Intermediate Value Theorem guarantees that there is at least one \( c \) in \((a, b)\).

(d) True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of \( f \).

**Solution:** False. There is nothing in the definition of horizontal asymptote that implies this.

(e) True or false. If \( f \) and \( g \) are differentiable, then \( \frac{d}{dx}[f(x)g(x)] = f'(x)g'(x) \).

**Solution:** False. Look up the product rule.

(f) True or false. If \( f \) and \( g \) are differentiable, then \( \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)}{g'(x)} \).

**Solution:** False. Look up the quotient rule.

(g) True or false. If \( f \) and \( g \) are differentiable and \( h(x) = f \circ g \), then \( h'(x) = f'(g(x))g'(x) \).

**Solution:** False. Look up the chain rule.
4. (40 points) Suppose the functions $f$ and $g$ have derivatives at all their domain points and their values at certain points are given in the table. The next four problems refer to these functions $f$ and $g$. Recall that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. In each case, a function $H(x)$ is given in terms of $f(x)$ and $g(x)$. You are asked to find $H'(x)$ at the value of $x$ provided.

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<tr>
<th>$x$</th>
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(a) The function $H$ is defined by $H(x) = f(x^2)$. Find $H'(1)$.

(A) 6  (B) 12  (C) 18  (D) 24  (E) 44

**Solution:** A. $H'(x) = f'(x^2) \cdot 2x$, so $H'(1) = f'(1) \cdot 2 = 6$.

(b) The function $J$ is defined by $J(x) = f(g(f(x)))$. Use the chain rule to find $J'(2)$.

(A) 6  (B) 9  (C) 12  (D) 21  (E) 48

**Solution:** E. $J'(x) = f'(g(f(x))) \cdot g'(f(x)) \cdot f'(x)$, so $J'(2) = f'(g(2)) \cdot g'(f(2)) \cdot f'(2) = f'(1) \cdot g'(5) \cdot f'(2) = 48$.

(c) The function $K$ is defined by $K(x) = g(x)/x^2$. Use the quotient and chain rules to find $K'(3)$.

(A) $-1/9$  (B) $1/3$  (C) $2/3$  (D) $4/9$  (E) $7/9$

**Solution:** D. Use the quotient rule to get $4/9$.

(d) The function $L$ is defined by $L(x) = (x + f(x))^{10}$. Use the chain and power rules to find $L'(0)$.

(A) 0  (B) $10 \cdot 2^9$  (C) $5 \cdot 2^9$  (D) $10 \cdot 2^{10}$  (E) $2^{11}$

**Solution:** D. $L'(x) = 10(x + f(x))^9 \cdot (1 + f'(x)) = 10 \cdot 2^9 \cdot 2$. 

3
(e) Use the information in the chart to find the y-intercept of the line tangent to the graph of \( f \) at the point \((2, 5)\).

\[ \text{Solution: A.} \ \text{The slope of the line tangent to the graph of} \ f \ \text{at the point} \ (2, 5) \ \text{is given in the table as 4, so the line is} \ y - 5 = 4(x - 2) \ \text{which has y-intercept} \ -3. \]

On all the following questions, **show your work.**

5. (20 points) Let \( k(x) = 2x^2 \).

(a) Using the definition of derivative, find \( k'(x) \)

**Solution:** The difference quotient is \( \frac{k(x + h) - k(x)}{h} = \frac{(2(x + h)^2 - 2x^2)}{h} = \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} = \frac{4xh + 2h^2}{h} \)

\( k'(x) = \lim_{h \to 0} \frac{4xh + 2h^2}{h} = 4x. \)

(b) Evaluate the function found above at \( x = 1 \) to find \( k'(1) \).

**Solution:** From above, \( k'(1) = 4. \)

(c) Use the information above to find an equation for the line tangent to the graph of \( k \) at the point \((1, k(1))\).

**Solution:** \( y - k(1) = y - 2 = 4(x - 1). \)

6. (20 points) A division of Moreken Industries manufactures microwave ovens. The daily cost (in dollars) of producing \( x \) ovens is given by \( C(x) = -0.03x^2 + 120x + 15000 \)

(a) What is the actual cost of producing the 201st microwave oven?

**Solution:** \( C(201) - C(200) = -0.03(201^2 - 200^2) + 120(201 - 200) + 5000 - 5000 = -12.03 + 120 = $107.97. \)

(b) Find the marginal cost function \( C'(x) \).

**Solution:** \( C'(x) = -0.03 \cdot 2x + 120 \)

(c) Find \( C''(200) \).

**Solution:** \( C''(200) = -12 + 120 = $108.00 \)

(d) Find the average cost function \( \overline{C}(x) \).

**Solution:** \( \overline{C}(x) = \frac{-0.03x^2 + 120x + 15000}{x} \).