Math 1120 Calculus Section 3 Test 4

April 24, 2002

Your name

All the problems are marked with their value. The total number of points available is 114. Throughout the test, show your work.

1. (10 points) Find the interval(s) over which the function \( f(x) = 2x^3 + 3x^2 - 36x + 17 \) is decreasing?

2. (10 points) Find the absolute maximum value of the function \( f(x) = e^{-x^2 + x} \) on the interval \(-2 \leq x \leq 3\).

3. (10 points) Let \( g(x) = \ln((2x - 3)(2x + 1)(x + 3)(x - 5)) \). Find the (implied) domain of \( g \).
4. (10 points) Sketch an example of a continuous function \( f(x) \) that has domain \([-4, 4]\), and satisfies the following requirements.

(a) \( f(-4) = f(-1) = f(2) = 0 \).
(b) \( f \) is increasing on \([-4, -2]\).
(c) \( f \) has a singular point at \( x = 3 \).
(d) \( f \) has a relative maximum at \( x = 3 \) and a value of 2 at \( x = 3 \).

Use the coordinate system given.

5. (10 points) Let \( f \) be the function whose graph is shown below. On the same axes, plot the graph of \( f'(x) \).

\[
f'(x) \approx .1(x^3 - 10x - 1)
\]
6. (10 points) Sketch an example of a function $f(x)$ that has domain $[-4, 4]$, and satisfies the following requirements. Please note: this problem has been slightly modified from the original, which interchanged the 1 and the 2 in the first two conditions.

(a) $\lim_{x \to -2^+} f(x) = 1$.

(b) $\lim_{x \to -2^-} f(x) = 2$.

(c) $f(2) = 0, f(0) = 3$

(d) $f$ is linear on the interval $[0, 4]$.

(e) $f$ has an absolute maximum at $x = 0$.

Use the coordinate system given.

7. (10 points) Solve the equation $2 + 3 \cdot 5^{2x+1} = 77$. 

8. (10 points) Compound Interest. Find the time required for an 8% investment compounded quarterly to triple.

9. (12 points) Compute the following limits.

(a) \( \lim_{x \to \infty} \frac{3x^3 - 5x^2 + 10}{2x^3 + 10x - 5} \).

(b) \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \).

(c) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \).
10. (12 points) Find the following derivatives.

(a) \( \frac{d}{dx} xe^x \)

(b) \( \frac{d}{dx} \frac{\ln(x)}{x} \)

(c) \( \frac{d}{dx} e^{\ln(x^5 + x^2 - 2x)} \)

11. (10 points) Let

\[
f(x) = \begin{cases} 
-x/2 + 2 & \text{if } x \leq -1 \\
x + 3 & \text{if } -1 < x < 3 \\
x^2 - 5x & \text{if } 3 \leq x 
\end{cases}
\]

Find an equation for the line tangent to the graph of \( f \) at the point \((4, -4)\).