1. (10 points) Find the interval(s) over which the function \( f(x) = 2x^3 + 3x^2 - 36x + 17 \) is decreasing?

Solution: Differentiate and factor to get \( f'(x) = 6x^2 + 6x - 36 = 6(x - 2)(x + 3) \), so the critical points are \( x = 2 \) and \( x = -3 \). Use the Test Interval Technique to solve the inequality \( f'(x) < 0 \). Only the interval \([-3, 2]\) works.

2. (10 points) Find the absolute maximum value of the function \( f(x) = e^{-x^2 + x} \) on the interval \(-2 \leq x \leq 3\).

Solution: Find \( f'(x) \) first and then the critical points that are between \(-2\) and \(3\). \( f'(x) = (-2x + 1)e^{-x^2 + x} \), so there is one critical point \( x = 1/2 \) and two endpoints to check: \( f(-2) = e^{-6} \approx 0.00247 \); \( f(3) = e^{-6} \approx 0.00247 \); and \( f(1/2) = e^{1/4} \approx 1.284 \), so the absolute maximum is \( f(1/2) \), which of course occurs at \( x = 1/2 \).

3. (10 points) Let \( g(x) = \ln((2x - 3)(2x + 1)(x + 3)(x - 5)) \). Find the (implied) domain of \( g \).

Solution: The domain of \( \ln z \) is the set \( z > 0 \) so we need to solve the inequality \((2x - 3)(2x + 1)(x + 3)(x - 5) > 0\). We do this by the Test Interval Technique. We get \((-\infty, -3) \cup (-1/2, 3/2) \cup (5, \infty)\).
4. (10 points) Sketch an example of a continuous function \( f(x) \) that has domain \([-4, 4]\), and satisfies the following requirements.

(a) \( f(-4) = f(-1) = f(2) = 0 \).
(b) \( f \) is increasing on \([-4, -2]\).
(c) \( f \) has a singular point at \( x = 3 \).
(d) \( f \) has a relative maximum at \( x = 3 \) and a value of 2 at \( x = 3 \).

**Solution:** Such a function is plotted below.

Use the coordinate system given.

![Graph of f(x)](image)

5. (10 points) Let \( f \) be the function whose graph is shown below. On the same axes, plot the graph of \( f'(x) \).

\[ f'(x) \approx .1(x^2 - 10x + 1) \]

![Graph of f'(x)](image)
6. (10 points) Sketch an example of a function \( f(x) \) that has domain \([-4, 4]\), and satisfies the following requirements. Please note: this problem has been slightly modified from the original, which interchanged the 1 and the 2 in the first two conditions.

(a) \( \lim_{x \to -2^+} f(x) = 1 \).

(b) \( \lim_{x \to -2^-} f(x) = 2 \).

(c) \( f(2) = 0, f(0) = 3 \).

(d) \( f \) is linear on the interval \([0, 4]\).

(e) \( f \) has an absolute maximum at \( x = 0 \).

Use the coordinate system given.

7. (10 points) Solve the equation \( 2 + 3 \cdot 5^{2x+1} = 77 \).

**Solution:** Subtract 2 from both sides to get \( 3 \cdot 5^{2x+1} = 75 \). Then divide by 3 to get \( 5^{2x+1} = 25 = 5^2 \). Since the bases are the same, the exponents must also be the same. Hence \( 2x + 1 = 2 \) and \( x = 1/2 \).
8. (10 points) Compound Interest. Find the time required for an 8% investment compounded quarterly to triple.

**Solution:** We need to solve the equation

\[ 3 = 1 \left( 1 + \frac{0.08}{4} \right)^{4t}. \]

Take logs of both sides to get \( 4t = \frac{\ln 3}{\ln 1.02} \). Divide both sides by four to get \( t = \frac{\ln 3}{4 \ln 1.02} \approx 13.87 \) years.

9. (12 points) Compute the following limits.

(a) \( \lim_{x \to \infty} \frac{3x^3 - 5x^2 + 10}{2x^3 + 10x - 5} \).

**Solution:** Since both polynomials have degree 3, the limit is the ratio of the leading coefficients, 3/2.

(b) \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \).

**Solution:** Factor the denominator and cancel the common terms to get \( \lim = 1/4 \).

(c) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \).

**Solution:** Rationalize the numerator (or factor the denominator) to get \( \lim = 1/6 \).
Math 1120  Calculus  Section 3  Test 4

10. (12 points) Find the following derivatives.

(a)  \( \frac{d}{dx} xe^x \)
   **Solution:** Use the product rule to get \( \frac{d}{dx} xe^x = e^x + xe^x \)

(b)  \( \frac{d}{dx} \ln(x) \)
   **Solution:** Use the quotient rule to get \( \frac{d}{dx} \frac{\ln(x)}{x} = \frac{1 - \ln(x)}{x^2} \)

(c)  \( \frac{d}{dx} e^{\ln(x^5 + x^2 - 2x)} \)
   **Solution:** The function is just the exponent since the two functions being composed are inverses of each other. Therefore, \( \frac{d}{dx} e^{\ln(x^5 + x^2 - 2x)} = 5x^4 + 2x - 2 \).

11. (10 points) Let
   \[
   f(x) = \begin{cases} 
   -x/2 + 2 & \text{if } x \leq -1 \\
   x + 3 & \text{if } -1 < x < 3 \\
   x^2 - 5x & \text{if } 3 \leq x 
   \end{cases}
   \]
   Find an equation for the line tangent to the graph of \( f \) at the point \((4, -4)\).
   **Solution:** Note that near \( x = 4 \), \( f'(x) = 2x - 5 \), so \( f'(4) = 8 - 5 = 3 \). Thus the line is given by \( y - 4 = 3(x + 4) \), which is equivalent to \( y = 3x + 16 \).