1. Write \( n = 1492 \) in the base \( b = 2 \) \( b = 4 \), and \( b = 8 \). What is the relationship among these representations? In other words, find a way to convert between these bases without translating them to and from decimal.

**Solution:** First compute the base 8 representation of 1492 to get 1492 = \( 2724_8 \). Then find the binary representation of each of the four digits 2,7,2, and 4. These are 010,111,010, and 100. Now you can string these representations together to get 010111010100. To get the base 4 representation of 10111010100, interpret each pair of binary digits as a base four digit, starting at the right. Thus 00 = 0; 01 = 1; 01 = 1; 11 = 3; 01 = 1 and 1 = 1. Thus 113110 is the base four representation of 1492.

2. Find the unknown digit \( x \) from the equation \( 2x_{3_4} = 1x_{10_3} \).

**Solution:** Interpret the two numbers to get \( 2x_{3_4} = 2 \cdot 4^2 + 4x + 3 \) and \( 1x_{10_3} = 1 \cdot 3^3 + 3^2x + 3 \). Then solve the resulting linear equation to find that \( x = 1 \).

3. Construct the tables of addition and multiplication for the base \( b = 6 \) and evaluate \( 215_6 + 304_6 \) and \( 203_6 \times 405_6 \).

**Solution:** (215)\(_6\) + (304)\(_6\) = 523\(_6\) and (203)\(_6\) \times (405)\(_6\) = 123423\(_6\)

4. Find the base \(-5\) representation of all the numbers from 1 to 25.

**Solution:** They are 1, 2, 3, 4, 140, 141, 142, 143, 144, 130, 131, 132, 133, 134, 120, 121, 122, 123, 124, 110, 111, 112, 113, 114, 100.

5. Find the base 4 representation of \( 1/9 \). Prove your answer.

**Solution:** \( 1/9 = 0.013_4 \) by the process of repeated multiplication. To prove this, let \( x = 0.013_4 \), and then note that \( 1000x = 13.013_4 \) in which case we have \( 1000x - x = 333_4x = 13_4 \), which is the same as the decimal equation \( 63x = 7 \), so \( x = 1/9 \).

6. Find the base \(-4\) representation of \( 1/3 \) and then show that your answer is correct.

**Solution:** \( 1.32_{-4} \). To see that this is correct, let \( x = 1.32_{-4} \). Then \( 100x = 132.32_{-4} \), and \( 100_{-4} - 1_{-4} = 114_{-4} \), so \( 113_{-4}x = 132_{-4} \). Translating these into decimal notation, we have \( 15x = 1 \cdot (-4)^2 + 3 \cdot (-4) + 2 = 5 \), so \( x = 1/3 \).

7. How many two-digit positive integers \( N \) have the property that the sum of \( N \) and the number obtained by reversing the order of the digits of \( N \) is a perfect square?
Solution: Let $N = 10x + y$. Then $10x + y + 10y + x = 11(x + y)$ must be a perfect square. Since $1 \leq x + y \leq 18$, it follows that $x + y = 11$. There are eight such numbers: 29, 38, 47, 56, 65, 74, 83, and 92.

8. A check is written for $x$ dollars and $y$ cents, both $x$ and $y$ two-digit numbers. In error it is cashed for $y$ dollars and $x$ cents, the incorrect amount exceeding the correct amount by $17.82$. Find a possible value for $x$ and $y$.

Solution: Note that $100y + x - (100x + y) = 99(y - x) = 1782$. It follows that $y - x = 18$, so any pair of two-digit numbers that differ by 18 will work.

9. The rightmost digit of a six-digit number $N$ is moved to the left end. The new number obtained is five times $N$. What is $N$?

Solution: The number is 142857. Let $N = \underline{abcdef}$ and let $x = \underline{abcde}$. Then we have $5N = 5(abcdef) = f\underline{abcde}$. Note that we have adopted the convention of underlining the digits of an integer in decimal notation. For example $\underline{ab} = 10a + b$. Note that $5(abcdef) = 5(\underline{abcde0} + f) = 5(10x + f) = f\underline{abcde} = 100000f + x$. This leads to $50x - x = 100000f - 5f$ or $49x = 99995f$. Factoring both sides gives $7^2x = 5 \cdot 7 \cdot 2857 \cdot f$, which is equivalent to $7x = 5 \cdot 2857 \cdot f$, from which it follows that $f = 7$. Therefore $x = 14285$ and $N = 142857$. 
