Throughout we use both the notations \( \binom{n}{r} \) and \( C_r^n \) for the number \( \frac{n!}{(n-r)!r!} \).

1. How many paths consisting of a sequence of horizontal and/or vertical line segments with each segment connecting a pair of adjacent letters in the diagram below, is the word CONTEST spelled out as the path is traversed from beginning to end?

   \[
   \begin{array}{cccccccc}
   & C & O & C & C & O & N & O & C \\
   & C & O & N & T & N & O & C & C \\
   & C & O & N & T & E & T & N & O & C \\
   & C & O & N & T & E & S & E & T & N & O & C \\
   & C & O & N & T & E & S & T & S & E & T & N & O & C \\
   \end{array}
   \]

2. Recall that a Yahtzee Roll is a roll of five indistinguishable dice.
   a. How many different Yahtzee Rolls are possible?
   b. How many Yahtzee Rolls have exactly 3 different numbers showing?

3. How many numbers can be expressed as a sum of three distinct members of the set \( \{4, 5, 6, 7, 8, 9, 10, 11, 12\} \)?

4. Let \( S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \).
   a. How many five element subsets does the set have?
   b. How many subsets of \( S \) have an odd number of members?
   c. How many subsets of \( S \) have 1 as a member?
   d. How many subsets have 1 as a member and do not have 2 as a member?

5. Imagine that the 4 \( \times \) 7 grid of squares below represents the streets of a part of the city where you live. You must walk 11 blocks to get from the lower left corner at A to the upper right corner at B.
   (a) How many different 11 block walks are there?
   (b) How many 11 block walks avoid the terrible corner marked with the bullet?
   (c) How many 11 block walks go through the terrible corner?
   (d) How many different 12 block walks are there from A to B?
   (e) How many different 13 block walks are there from A to B?
6. How many four-digit numbers have the property that the sum of the first three digits is the fourth digit. For example 1247 has the property.

7. How many numbers in the set \(\{100, 101, 102, \ldots, 999\}\) have a sum of digits equal to 9? B. How many four digit numbers have a sum of digits 9? C. How many integers less than one million have a sum of digits equal to 9?

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