Material from Lectures 4 through 6

1. Recurrence Relations

(a) Translate among the three definitions of sequence: listing, recursion, closed form. In other words, work problems like number 1 in (newly revised) homework 5.

(b) Find the characteristic equation of a first, second, or third order recurrence relation.

(c) Solve first and second order characteristic equations to find the general solutions of a recurrence relations.

(d) Use the initial values to produce a specific solution from the general solution.

(e) Prove that you answer is correct by mathematical induction.

2. Congruence and divisibility properties.

(a) Divisibility by 3 and 9

(b) Divisibility by 11

(c) Determining remainders when exponential expressions are divided by small numbers.

3. Cardinality, equivalence of sets

(a) Know definitions of one-to-one, onto, and bijection, and to recognize and construct examples of each.

(b) Be able to prove that a given function is one-to-one or onto.

(c) Be able to reproduce the proof of the equivalence of the following pairs of sets: $\mathbb{Z}^+ \approx \mathbb{Z}$, $\mathbb{Q} \approx \mathbb{Z}$, $[0, 1] \approx [0, 1] \times [0, 1]$, and $[0, 1) \approx [0, 1]$ where $\mathbb{Q}$ denote the rational numbers, $\mathbb{Z}$ the integers, and $\mathbb{Z}^+$ the positive integers.

(d) Prove the uncountability of the real number interval $[0, 1]$ using Cantor’s diagonalization procedure.

4. Set Arithmetic

(a) Understand and use the set identities on page 7 of Lecture 6, including recognizing by name the identities.

(b) Use characteristic functions to prove set identities.

(c) Use Venn diagrams.
(d) Understand and use the principle of Inclusion-Exclusion (see problem set 6).

The material below will be covered on the final exam.

5. Combinatorics

   (a) Understand and use the addition rule and the multiplication rule.

   (b) Understand and use the principle of Inclusion-Exclusion (see problem set).

   (c) Use permutations and combinations to count sets of arrangements with repetition and without repetitions and to count selections with and without repetition (see problem set).

6. Probability

   (a) Definitions of Sample space, Event, Outcome, Equally likely probability measure.

   (b) Understand and use principles of combinatorics to compute probabilities.

   (c) Use Pascal’s triangle (binomial coefficients) to find probabilities associated with binomial experiments.