February 14, 2000

Your name ________________________________

It is important that you show your work. The total value of this test is 110 points.

1. (10 points) Find the base -6 representation of 29.

2. (20 points)
   
   (a) Use the division algorithm to find the unique integers $r$ and $q$ satisfying
       
       $297 = 73q + r$ and $0 \leq r < 73$.

   (b) Solve the decanting problem for containers of sizes 73 and 297; that is find integers $x$ and $y$ satisfying
       
       $73x + 297y = d$

       where $d$ is the GCD of 73 and 297.
3. (10 points) Find the base 6 representation of each of the following:

(a) 247

(b) $8\frac{13}{36}$

(c) 0.15

4. (15 points)

(a) Construct the base 6 addition table and the base 6 multiplication table.

(b) Use the tables in (a) to carry out the multiplication $1051_6 \times 204_6$.

(c) Convert the three numbers $1051_6$, $204_6$, and your answer in (b) to their decimal equivalents and carry out the multiplication in decimal representation to check your answer to (b).
5. (20 points) Notice that

\[
\begin{align*}
2 &= 2 = 2 \cdot 1 \quad &\text{(1)} \\
2 + 4 &= 6 = 3 \cdot 2 \quad &\text{(2)} \\
2 + 4 + 6 &= 12 = 4 \cdot 3 \quad &\text{(3)} \\
2 + 4 + 6 + 8 &= 20 = 5 \cdot 4 \quad &\text{(4)}
\end{align*}
\]

(a) List the next three equations suggested by the pattern.

(b) Given that the four equations above are the 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, and 4\textsuperscript{th}, write the \(n\textsuperscript{th}\) equation of the sequence.

(c) Use mathematical induction to prove that the \(n\textsuperscript{th}\) equation is true for all positive integer values of \(n\).
6. (15 points) Divisors and Prime factorization

(a) Find two different numbers \(a\) and \(b\) both of which are multiples of 6 such that each one has exactly 8 positive integer divisors.

(b) Find the greatest common divisor (GCD) of your \(a\) and \(b\).

(c) Find the least common multiple (LCM) of your \(a\) and \(b\).

7. (20 points) Compute the remainders when each \(n\) below is divided by the given \(d\).

(a) \(n = 3^{2001}\), \(d = 10\)

(b) \(n = 5^{2001}\), \(d = 7\)

(c) \(n = 123,456,789,012,345,678\), \(d = 9\)

(d) \(n = 123,456,789,012,345,678\), \(d = 11\)