1. (10 points) Find the base 6 representation of 2000.

2. (10 points) Find the base 6 representation of $\frac{1}{5}$.

3. (10 points) Find a pair of relatively prime integers $m$ and $n$ for which $\frac{m}{n} = 1.2\overline{3}$. Two numbers are relatively prime if their greatest common divisor is 1.

4. (10 points) Find a base 7 digit $d$ such that $2d16_8 = d405_7$. 

5. (10 points) Find the best (winning) move in the game of Bouton’s Nim (17, 13, 12, 11).

6. (12 points) Let $M = 161,161$ and let $N = 12,376$.
   
   (a) Compute $LCM(M, N)$

   (b) Compute $GCD(M, N)$

   (c) Find the number of divisors of $M$. 
7. (20 points) Look at the four equations below.

\[
\begin{align*}
2 &= 2 \cdot 1 \\
2 + 4 &= 3 \cdot 2 \\
2 + 4 + 6 &= 4 \cdot 3 \\
2 + 4 + 6 + 8 &= 5 \cdot 4
\end{align*}
\]

a. Write the next three equations in the sequence.

b. If the four equations above correspond to \( k = 1, 2, 3, \) and 4, what is the nth equation?

c. Prove by mathematical induction that the nth equation is true for all integers \( n \geq 1. \)

8. (10 points) Find the representations of the integers 1 through 13 in base \(-6\).
9. (15 points) Solve the equation $123x + 456y = 3$ for integers $x$ and $y$.

10. (13 points) Prove that $2^n \leq n!$ for all integers $n \geq 4$. 