1. (20 points) Let $P$ denote the compound proposition defined by $P : (p \rightarrow q) \rightarrow r$ and let $Q : p \rightarrow (q \rightarrow r)$. Test the associativity of $\rightarrow$ by determining whether $P$ and $Q$ are logically equivalent.
2. (24 points) Determine the truth value of the following statements if the universe of discourse of each variable is the set of real numbers.

1. $\forall x \exists y (x = y^2)$

2. $\forall x \exists y ((x + y = 2) \land (2x - y = 1))$

3. $\exists x (x^2 = -1)$

4. $\forall x \neq 0 \exists y (xy = 1)$

5. $\exists x \exists y (x + y \neq y + x)$

6. $\forall x \exists y (x + y = 1)$

7. $\exists x (x^2 = 2)$

8. $\exists x \forall y \neq 0 (xy = 1)$

9. $\forall x \forall y \exists z (z = \frac{x + y}{2})$

10. $\exists x \exists y ((x + 2y = 2) \land (2x + 4y = 5))$

11. $\exists x \forall y (xy = 0)$

12. $\forall x \exists y (x^2 = y)$
3. (20 points) Let $D$ denote the set of all real numbers and let $P$ and $Q$ denote
the two-place predicates on $D$ defined by $P(x, y) : x \leq y$ and $Q(x, y) : y \leq x$.
Find the truth value of each of the compound propositions.

(a) $\forall x \forall y (P(x, y) \land Q(x, y)) \rightarrow x = y$.

(b) $\forall x \forall y (P(x, y) \rightarrow \neg Q(x, y))$.

(c) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$.

(d) $\forall x \forall y (P(x, y) \lor Q(x, y)) \rightarrow x \neq y$.

(e) $\forall x \forall y x \neq y \leftrightarrow (\neg P(x, y) \lor \neg Q(x, y))$. 

\[ \]
4. (20 points) Notice that

\[
\begin{align*}
\frac{1}{1 \cdot 2} &= \frac{1}{2} & (1) \\
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} &= \frac{2}{3} & (2) \\
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} &= \frac{3}{4} & (3)
\end{align*}
\]

(a) List the next two equations suggested by the pattern.

(b) Given that the three equations above are the 1st, 2nd, and 3rd, write the nth equation of the sequence.

(c) Use mathematical induction to prove that the nth equation is true for all positive integer values of n.
5. (20 points) Prove that $4^n - 1$ is divisible by 3 for all $n \geq 1$.

6. (10 points) In a group of 100 students, the following facts are known:

- 50 take accounting,
- 40 take biology,
- 35 take chemistry,
- 12 take both accounting and biology,
- 10 take accounting and chemistry,
- 11 take chemistry and biology, and
- 5 take all three subjects.

How many take none of the three subjects?
7. (20 points) Let $\mathbb{Z}$ denote the set of all integers. Classify each of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ as one-to-one and onto, one-to-one and not onto, onto and not one-to-one, neither onto nor one-to-one. Prove your answers.

(a) Let $f(n) = \begin{cases} 
2n & \text{if } n \geq 0 \\
-2n - 1 & \text{if } n < 0 
\end{cases}$

(b) Let $f(n) = \begin{cases} 
n - 1 & \text{if } n \geq 1 \\
n + 1 & \text{if } n \leq 0 
\end{cases}$

(c) $f(n) = -n$

(d) $f(n) = |n|$