April 3, 1998

In the first 4 problems each part counts 6 points each for a total of 6 \cdot 6 = 36 points and the final 3 problems count as marked.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Let \( L(x) \) be the linearization of the function \( f(x) = \sqrt{x + 4} \) at the point \( a = 0 \). What is \( L(0.5) \)?

   (A) 1.95   (B) 2.13   (C) 2.25   (D) 2.5   (E) 2.55

2. For how many points on the curve \( x^2 + 2y^2 = 1 \) does the tangent line have slope 1?

   (A) 0   (B) 1   (C) 2   (D) 4   (E) more than 4

3. The \( x \)-coordinate of one point on the curve \( x^2 + 2y^2 = 4 \) where the tangent line has slope -1 is (approximately, to three decimal places)

   (A) 1.234   (B) 1.347   (C) 1.546   (D) 1.633   (E) 1.668
4. Suppose the functions $f$ and $g$ are given completely by the table of values shown. The next four problems refer to the functions $f$ and $g$ given in the tables.

\begin{tabular}{|c|c|c|c|c|}
\hline
$x$ & $f(x)$ & $f'(x)$ & $x$ & $g(x)$ & $g'(x)$ \\
\hline
0 & 2 & 1 & 0 & 5 & 5 \\
1 & 7 & 3 & 1 & 7 & 3 \\
2 & 5 & 4 & 2 & 4 & 4 \\
3 & 1 & 2 & 3 & 2 & 6 \\
4 & 3 & 3 & 4 & 6 & 10 \\
5 & 6 & 4 & 5 & 3 & 4 \\
6 & 0 & 5 & 6 & 1 & 2 \\
7 & 4 & 1 & 7 & 0 & 1 \\
\hline
\end{tabular}

(a) The function $h$ is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(2)$.

\begin{itemize}
  \item[(A)] 1
  \item[(B)] 4
  \item[(C)] 6
  \item[(D)] 10
  \item[(E)] 12
\end{itemize}

(b) The function $k$ is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(3)$.

\begin{itemize}
  \item[(A)] 1
  \item[(B)] 4
  \item[(C)] 6
  \item[(D)] 10
  \item[(E)] 12
\end{itemize}

(c) The function $H$ is defined by $H(x) = f(x)/g(x)$. Use the quotient rule to find $H'(4)$.

\begin{itemize}
  \item[(A)] $-12$
  \item[(B)] $-1/3$
  \item[(C)] $-1/2$
  \item[(D)] 3/10
  \item[(E)] 1
\end{itemize}
5. (25 points)

(a) Find \( \frac{d}{dx}(\tan x) \)

(b) Write an equation involving \( \tan \), \( \arctan \), the composition operation, and the identity function.

(c) Differentiate both sides of the equation in (b).

(d) Use the result in (c) to find an expression for \( \frac{d}{dx}(\arctan x) \).
6. (20 points)

(a) Find $\frac{dy}{dx}$ at the point $(-1, 1)$ if $x$ and $y$ are related by

$$x^2 + xy - y^3 = xy^2$$

(b) Use the information in (a) at find an equation for the line tangent to the curve

$$x^2 + xy - y^3 = xy^2$$

at the point $(-1, 1)$. 
7. (28 points)

(a) Let $V(t)$ be the volume of a cube with an edge of length $x(t)$. Find $dV/dt$ in terms of $dx/dt$.

(b) Let $V(t)$ be the volume of a sphere with a radius $r(t)$. Find $dV/dt$ in terms of $dr/dt$.

(c) Let $V(t)$ be the volume of a cube with surface area of $S(t)$. Find $V(t)$ as a function of $S(t)$. Find $dV/dt$ in terms of $dS/dt$.

(d) Referring to part (c), suppose the surface area is changing at the rate $24\text{in}^2/\text{sec}$ precisely at the time when the volume is 1728 cubic inches. How fast is the volume changing at this moment?