March 26, 2004 Name

There are 135 points available on this test. Each question is marked with its value. To get full credit for a problem, you must show your work. Correct answers with incorrect supporting work will receive substantially reduced credit.

1. (15 points) Let \( p(x) = x^2 - 4x + 5 \).
   
   (a) Compute \( p'(x) \)

   (b) Compute \( p''(x) \)

   (c) Use the information in (a) to find an equation for the line tangent to the graph of \( p \) at the point \((1, 2)\).

2. (20 points) Consider the astroid \( x^{2/3} + y^{2/3} = 4 \).
   
   (a) Show that the point \((-3\sqrt{3}, 1)\) belongs to the graph.

   (b) Find \( y' \) as a function of \( x \) and \( y \) using implicit differentiation.

   (c) Find the slope of the line tangent to the curve at the point \((-3\sqrt{3}, 1)\).

   (d) Find an equation for the tangent line whose slope you found above.
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3. (30 points) Suppose the functions $f$ and $g$ are given partially by the table of values shown. The next problems refer to the functions $f$ and $g$ given in the tables. Consider the table of values given for the functions $f$, $f'$, $g$, and $g'$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
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<td>6</td>
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<tr>
<td>2</td>
<td>5</td>
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<td>1</td>
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<tr>
<td>3</td>
<td>6</td>
<td>4</td>
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<td>6</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Let $K(x) = f \circ g(x)$. Compute $K'(3)$

(b) Let $L(x) = f(x) \cdot g(x)$. Compute $L'(2)$.

(c) Let $U(x) = f \circ f(x)$. Compute $U'(1)$.

(d) Let $V(x) = g(x)/f(x)$. Compute $V'(4)$.

(e) Let $W(x) = (g(x))^2$. Compute $W'(5)$.

(f) Let $Z(x) = g(x^2 \cdot f(x))$. Compute $Z'(1)$.
4. (25 points)

(a) Find \( \frac{d}{dx}(\sin x) \)

(b) Write an equation involving the functions \( \sin \) and \( \sin^{-1} \), the composition operation, and the identity function. In other words write an equation that shows you know what \( \sin^{-1} x \) is.

(c) Differentiate both sides of the equation in (b).

(d) Use the result in (c) to find an expression for \( \frac{d}{dx}(\sin^{-1} x) \).

(e) Let \( h(x) = \sin^{-1}(x^2) \). Compute \( h'(x) \).
5. (25 points) Compute the following derivatives.

(a) \( \frac{d}{dx} e^{\sin x} \)

(b) \( \frac{d}{dx} \ln(\tan x) \)

(c) \( \frac{d}{dx} \sqrt{x} \ln x \)

(d) \( \frac{d}{dx} (\cos(x^2))^3 \)

(e) \( \frac{d}{dx} \tan^{-1}(2x) \)
6. (20 points) Suppose \( f \) is defined by:

\[
    f(x) = \begin{cases} \ln(3x) & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}
\]

(a) Find \( f'(3) \).

(b) Find \( f'(-e) \).

(c) Find an equation for the line tangent to the graph of \( f \) at the point \((-e, f(-e))\).