1. (10 points) Suppose \( \int_{-8}^{-5} f(x)dx = 2 \), \( \int_{-8}^{-7} f(x)dx = 10 \), \( \int_{-6}^{-5} f(x)dx = 8 \).

(a) Find \( \int_{-7}^{-6} f(x)dx = \)

**Solution:** Using property 5 of integrals, note that
\[
\int_{-7}^{-6} f(x)dx = \int_{-7}^{-8} f(x)dx + \int_{-8}^{-6} f(x)dx
\]
\[
= -10 + 2 + (-8) = -16.
\]

(b) \( \int_{-6}^{-7} (2f(x) - 10)dx = \)

**Solution:** Using what we learned above and the linearity of the integral,
\[
\int_{-6}^{-7} (2f(x) - 10)dx = -2(-16) + 10(-6 - (-7)) = 42.
\]

2. (15 points) Given
\[
f(x) = \int_0^x \frac{t^2 - 4}{1 + \cos^2(t)} dt
\]
At what value of \( x \) does the local max of \( f(x) \) occur?

**Solution:** By FTC, the derivative of \( f \) is \( f'(x) = \frac{x^2 - 4}{1 + \cos^2(x)} \), which is positive on \( (-\infty, -2) \cup (2, \infty) \). So \( f \) is increasing to the left of \(-2\) and decreasing to the right of \(-2\). Therefore it has a local (relative) maximum at \(-2\). The other extremum is at \( x = 2 \) where \( f \) has a local minimum.
3. (24 points) Find the following indefinite integrals.

(a) \[ \int \cos^3 \theta \sin^3 \theta \, d\theta \]

**Solution:** Let \( u = \sin \theta \), then \( du = \cos \theta \, d\theta \). Replace \( \cos^2 \theta \) with \( 1 - \sin^2 \theta \) and the integral becomes \( \int u^3 - u^5 \, du \). Thus \( \int \cos^3 \theta \sin^3 \theta \, d\theta = \frac{\sin^4 \theta}{4} - \frac{\sin^6 \theta}{6} + C \).

(b) \[ \int (x - 1)^2 \, dx \]

**Solution:** Since \( (x - 1)^2 = x^2 - 2x + 1 \), the integral is simply \( \frac{x^3}{3} - x^2 + x + C \).

(c) \[ \int \frac{2x}{x^2 + 1} \, dx \]

**Solution:** Let \( u = x^2 + 1 \). Then \( du = 2x \, dx \) and \( \int \frac{2x}{x^2 + 1} \, dx = \int \frac{u}{u} \, du = \ln |u| + C = \ln(x^2 + 1) + C \).

(d) \[ \int \frac{1}{\sqrt{4 - x^2}} \, dx \]

**Solution:** Let \( x = 2 \sin \theta \). Then \( dx = 2 \cos \theta \, d\theta \) and \( 4 - x^2 = 4 - 4 \sin^2 \theta = 4 \cos^2 \theta \) which is nonnegative for \( 0 \leq \theta \leq \pi/2 \). It follows that \( \int \frac{1}{\sqrt{4 - x^2}} \, dx = \int 1 \, d\theta = \theta + C = \sin^{-1}(x/2) + C \).
4. (64 points) Use the evaluation theorem as needed to find each of the definite and improper integrals below. Each improper integral must be identified as such to get credit.

(a) \( \int_0^2 \frac{d}{dx}[(x^2 - 3)(x^3 - 1)] \, dx \)

**Solution:** On course the integral of the derivative is just the growth of the function, so

\[
\int_0^2 \frac{d}{dx}(x^2 - 3)(x^3 - 1) \, dx = (x^2 - 3)(x^3 - 1)|_0^2 = (4 - 3)(8 - 1) - (-3)(-1) = 7 - 3 = 4.
\]

(b) \( \int_4^9 \frac{9}{\sqrt{x}} \, dx \)

**Solution:** Just use the power rule to get

\[
9 \cdot 2 \cdot x^{1/2} |_4^9 = 18(3 - 2) = 18.
\]

(c) \( \int_0^{\pi/2} \cos x \cos(\sin x) \, dx \)

**Solution:** Let \( u = \sin x \). Then \( du = \cos x \, dx \). Therefore,

\[
\int_0^{\pi/2} \cos x \cos(\sin x) \, dx = \int \cos x \cos(u) \, du = \sin(\sin(x))|_0^{\pi/2} = \sin 1 - \sin 0 \approx 0.84147. \]

Be sure the calculator is in radian mode for this calculation.

(d) \( \int_3^4 \frac{x - 1}{x^2 - 4} \, dx \)

**Solution:** Use partial fractions to decompose the integrand as follows:

\[
\frac{x - 1}{x^2 - 4} = \frac{x - 1}{(x - 2)(x + 2)} = \frac{A}{x - 2} + \frac{B}{x + 2}.
\]

Solve for \( A \) and \( B \) to get \( A = 1/4 \) and \( B = 3/4 \). Therefore we have

\[
\int_3^4 \frac{1}{4(x - 2)} + \frac{3}{4(x + 2)} \, dx.
\]

Then anti-differentiate to get

\[
(1/4) \ln(x - 2) + (3/4) \ln(x + 2)|_3^4 = (1/4) \ln 2 + (3/4) \ln 6 - (3/4) \ln 5 \approx 0.3100.
\]
\[(e) \int_\infty^\infty (x \ln x)^{-1} \, dx\]

**Solution:** Use the substitution \( u = \ln x \). Then \( \int (x \ln x)^{-1} \, dx = \ln |u| = \ln(|\ln |x||)) \), so \( \int_\infty^\infty (x \ln x)^{-1} \, dx = \lim_{t \to \infty} \ln(\ln |x|)|_e^t = \lim_{t \to \infty} \ln(\ln t) \), which diverges because \( \ln t \) is unbounded.

\[(f) \int_1^4 |x^3 - 6x^2 + 11x - 6| \, dx \). Note that \( f(x) = x^3 - 6x^2 + 11x - 6 \) factors into \((x - 1)(x - 2)(x - 3)\).

**Solution:** Break the integral into three integrals, \( \int_1^2, \int_2^3, \int_3^4 \). Note that \( f(x) \) is positive over the first and last of these and negative over the second one. Therefore, \( \int_1^4 |x^3 - 6x^2 + 11x - 6| \, dx = \int_1^2 f(x) - \int_2^3 f(x) + \int_3^4 f(x) = F(2) - F(1) - (F(3) - F(2)) + F(4) - F(3) = 0.25 + 0.25 + 2.25 = 2.75 \), where \( F(x) = x^4/4 - 2x^3 + 11x^2/2 - 6x \).

\[(g) \int_0^1 x(x - 2)^9 \, dx\]

**Solution:** Let \( u = x - 2 \). Then \( du = dx \), \( x = u + 2 \) and our integral can be written \( \int_0^1 (u + 2)u^9 \, du \) which is just \( (u^{11}/11 - 2u^{10})|_2^{-1} = \frac{22 - 10 - 2^{11}}{110} \approx -18.509 \).

\[(h) \int_0^1 \frac{1}{\sqrt{x}} \, dx\]

**Solution:** This integral is improper since the integrand has a vertical asymptote at \( x = 0 \). Thus we have \( \int_0^1 \frac{1}{\sqrt{x}} \, dx = \lim_{t \to 0^+} \int_0^t \frac{1}{\sqrt{x}} \, dx = \lim_{t \to 0^+} 2x^{1/2}|_t^1 = \lim_{t \to 0^+} 2 - 2t^{1/2} = 2.\)
5. (15 points) Construct a triangle with an acute angle $\theta$ such that $\tan \theta = x/2$. Then compute each of the following in terms of $x$.

(a) $\sin \theta$

Solution: $\sin \theta = \frac{x}{\sqrt{x^2 + 2^2}}$

(b) $\sin(2\theta)$

Solution: $\sin(2\theta) = 2 \sin \theta \cos \theta = 2 \frac{x}{\sqrt{x^2 + 2^2}} \cdot \frac{2}{\sqrt{x^2 + 2^2}} = \frac{4x}{x^2 + 2^2}$.

(c) $\csc \theta$

Solution: $\csc \theta = \frac{\sqrt{x^2 + 2^2}}{x}$.

6. (15 points) For each integral below, use the substitution $\theta$ such that $x = 2 \tan \theta$ to find an equivalent $d\theta$ integral. Do not evaluate.

(a) $\int_{0}^{1} \frac{x^2}{\sqrt{4 + x^2}} \, dx$

Solution: $\int_{0}^{1} \frac{x^2}{\sqrt{4 + x^2}} \, dx = \int_{0}^{\pi/4} \frac{4 \tan^2 \theta \cdot 2 \sec^2 \theta}{2 \sec \theta} \, d\theta$.

(b) $\int \frac{1}{4 + x^2} \, dx$

Solution: $\int \frac{1}{4 + x^2} \, dx = \frac{1}{2} \int 1 \, d\theta$.

(c) $\int x \sqrt{4 + x^2} \, dx$

Solution: $\int x \sqrt{4 + x^2} \, dx = \int 4 \tan \theta \cdot 2 \sec \theta \cdot 2 \sec^2 \theta d\theta = 8 \int \tan \theta \cdot \sec^3 \theta d\theta$. 
