1. (5 points) What is the limit of the sequence $a_n$ defined by $a_n = \frac{n^2 - 6n + 5}{3n^2 + 2n - 12}$, as $n \to \infty$?

2. (15 points) Consider the series $\sum_{n=1}^{\infty} a_n$ where

$$a_n = \frac{(6n^2 + 4)3^{n+1}}{5^n}$$

Does the series converge absolutely, converge conditionally or diverge. What test did you use? Why is it conclusive?

3. (15 points) Consider the series $\sum_{n=1}^{\infty} a_n$ where

$$a_n = \frac{e^{n-3}}{\sqrt{n + 5(n + 2)!}}$$

Does the series converge absolutely, converge conditionally or diverge. What test did you use? Why is it conclusive?
4. (20 points) Find the interval of convergence for the given power series.

\[ \sum_{n=1}^{\infty} \frac{(x - 8)^n}{n(-6)^n} \]

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?

5. (20 points) Consider the power series

\[ \sum_{n=1}^{\infty} \frac{(7x)^n}{n^{11}} \]

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?
6. (20 points) Find all the values of $x$ such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$$

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?
7. (20 points) Suppose that the Maclaurin series for \( f(x) = \frac{5x}{(7 + x)} \) is \( \sum_{n=0}^{\infty} c_n x^n \).

Find the coefficients, \( c_0, c_1, c_2, c_3, \) and \( c_4 \). Find the radius of convergence \( R \) of the power series.

8. (10 points) Find the Maclaurin series of the function \( f(x) = (9x^2)e^{-x} \). That is, find \( c_0, c_1, \) etc. so that \( f(x) = \sum_{n=0}^{\infty} c_n x^n \). Hint: first find the series for \( f(x) = e^{-x} \).
9. (10 points) Find the Maclaurin series of the function \( f(x) = (7x^2) \sin(x) \). That is, find \( c_0, c_1, \) etc. so that \( f(x) = \sum_{n=0}^{\infty} c_n x^n \).

10. (20 points) Suppose \( \sum_{n=0}^{\infty} c_n (x - 2)^n \) converges at \( x = 3 \).

   (a) Must it converge for \( x = 0 \)?

   (b) Must it converge for \( x = 1 \)?

   (c) Must it converge for \( x = 1.2 \)?

   (d) What can be said about the radius of convergence \( R \)?

   (e) What can be said about the convergence of \( \sum_{n=0}^{\infty} nc_n (1.5 - 2)^n \)? How is this series related to the one above?
11. (20 points) Find the Taylor polynomial $T_5(x)$ for $f(x) = \sin(2x)$ at $x = \pi$.

b. Find an upper bound for $|R_5(x)|$ on the interval $[\pi/2, 3\pi/2]$.

c. Find the radius of convergence of the Taylor series.