1. Is it possible to pair integers 1, 2, ..., 50, so that all sums of pairs should be different primes?

2. Some marbles in a bag are red and the rest are blue. If one red marble is removed, then one-seventh of the remaining marbles are red. If two blue marbles are removed instead of one red, then one-fifth of the remaining marbles are red. How many marbles were in the bag originally?

3. Is the number $2.\overline{9} = 2.9999\ldots$ the same as 3? Explain why or why not.

4. Are the numbers getting bigger or smaller: $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$?

5. Seventeen people attend a party at which each person shakes hands with all the rest. How many handshakes take place?

6. Find a positive integer bigger than 30 which is expressible as the sum of three distinct primes in exactly one way. Prove your answer.

7. Suppose $r$ is a positive root of $x^2 - 7x = 1$. Find the distance between $r$ and its reciprocal.

8. How many different positive amounts can be made using one or more coins from a stack consisting of a penny, a nickel, a dime, a quarter, and a half-dollar?

9. The sum of two numbers is 7 and their product is 8. Find the sum of their reciprocals.

10. Suppose you had two pennies, three nickels, four dimes and five quarters. How many different amounts could you make?

11. When 10 pounds of cucumbers were bought at a market, they contained 99% water. After several days they dried out to 98% water. How many pounds of water do they contain now?

12. Baby Jill is too young to step on the scales by herself, so the nurse holds her on the scales, and together they weigh 160 pounds. Then Jill’s father gets on the scales holding Jill and together they weigh 185 pounds. Finally, Jill’s father holds the nurse on the scales, and together they weight 280. How much does baby Jill weigh?

13. It took Huck Finn 5 days to go downstream on the Mississippi by ship, and 7 days to make the same trip upstream, again on the ship. How long will it take him to make the trip downstream on a raft?
14. Complete the following sentence by spelling out a number which makes the sentence correct.
This sentence consists of ________________ letters.

15. Pick two positive integers and generate the following sequence. The first integer picked is the first number and the second is the second. The third is the quotient of 1 plus the second and the first. Then get the fourth by doing the same thing with the third and second. Continue the process, getting the fifth, sixth, etc. For example, suppose the first number is 3 and the second is 5. Then the third would be \( \frac{1+5}{3} = 2 \) and the fourth would be \( \frac{2+1}{5} = \frac{3}{5} \), and the fifth is \( \frac{3+1}{2} = \frac{4}{5} \). Now compute the sixth number in the sequence:

\[
\frac{4}{5} + \frac{1}{3} = 3
\]

and the one following that is:

\[
\frac{3}{5} + 1 = 5,
\]

so you can see that the sequence starts all over again. Such sequences are called *periodic*. This one has period 5 because the sixth term is the same as the first, etc. Repeat the process with two other initial picks. Again you get periodicity. Do you always get a periodic sequence?

16. There are four positive integers \( n \) such that \( n^2 + 105 \) is a perfect square. What is the sum of these four integers?

17. Find the value of the expression

\[
1^2 - 2^2 + 3^2 - 4^2 + \cdots - 1998^2 + 1999^2.
\]