1. The date for Easter Sunday in year $y$ can be computed as follows: Let $a = y \mod 19, b = y \mod 4, c = y \mod 7, d = (19a + 24) \mod 30, e = (2b + 4c + 6d + 5) \mod 7, \text{ and } r = (22 + d + e)$. If $r \leq 31$, then Easter Sunday is March $r$; otherwise it is April $[r \mod 31]$. Compute the date for Easter Sunday in the years 1999, 2000, and 2001.

2. Here is a second method for computing the date of Easter Sunday in a given year $N$. Recall that $n \div d = \lfloor n/d \rfloor$. Let $a = N \mod 19, b = N \div 100, c = N \mod 100, d = b \div 4, e = b \mod 4, f = (b+8) \div 25, g = (b-f+1) \div 3, h = (19a+b-d-g+15) \mod 30, i = c \div 4, j = c \mod 4, k = (32+2e+2i-h-j) \mod 7, l = (a+11h+22k) \div 451, m = (h+k-7l+114) \div 31, \text{ and } n = (h+k-7l+114) \mod 31$. Then Easter Sunday falls on the $n$th day of the $m$th month of the year. Compute the dates for Easter Sunday for the same years as in problem 1.

3. Find all solutions of $2^n + 7 = x^2$ in which both $n$ and $x$ are integers.

4. Prove that among 18 consecutive three digit numbers, there must be at least one which is divisible by the sum of its digits.

5. Six dice are strung on a rigid wire (the wire passes through two opposite faces of each die). Each die can be rotated independently of the others. Prove that it is always possible to rotate the dice and then place the wire horizontally on a table in such a way that the six-digit number formed by their top faces is divisible by 7. (The faces of a die are numbered from 1 to 6; the sum of the numbers on opposite faces is always equal to 7.)

6. Let $f(x) = 375x^5 - 131x^4 + 15x^2 - 435x - 2$. Find the remainder when $f(97)$ is divided by 11.

7. Prove that a number is divisible by 3 if and only if the sum of its digits is divisible by 3, and that an integer is divisible by 9 if and only if the sum of its digits is divisible by 9.

8. Given the number 2492, double the units digit and subtract it from the number formed by the other digits. We get $249 - 2 \times 2 = 245$. Repeating this algorithm we get $24 - 2 \times 5 = 14$. Since 14 is clearly divisible by 7, the original number 2492 must be divisible by 7. Prove this rule for checking divisibility by 7.

9. If $a$ and $b$ are odd integers, prove that $a^2 + b^2$ is never a square.