Definitions and Basic Concepts

Ordinary differential equation (ODE), initial value problem (IVP)
Solution of an ODE and of an IVP
IVP/integral equation equivalence
Autonomous/non-autonomous distinction
Phase portrait
Exponential of a matrix/operator
Topological equivalence in phase portraits
Rest point/critical point/equilibrium, periodic orbit/cycle
Stability/asymptotic stability in the sense of Lyapunov, Lyapunov functions
Invariant sets
Stable/unstable manifold
Hyperbolicity of singularities
Index of an isolated singularity
Alpha- and omega-limit sets

Statements of Theorems

Existence/Uniqueness for linear IVP’s $\dot{x} = Ax, \ x(t_0) = x_0$ ($A$ an $n \times n$ matrix)
Existence/Uniqueness for $\dot{x} = f(x, t), \ x(t_0) = x_0$ ($x \in \mathbb{R}^n$)
Continuous dependence of solutions on initial conditions
Behavior of solutions near an endpoint of the maximal interval of existence
Lyapunov stability theorems
Flowbox Theorem
Hartman-Grobman Theorem
Stable Manifold Theorem
Cycles must surround singularities (and indices sum to 1)
Bendixson’s Criterion/Dulac’s Criterion
Poincaré-Bendixson Theorem
Poincaré Annular Region Theorem

Problems/Exercises

Classification of singularities of linear and nonlinear systems
Determining stability using Lyapunov functions
Identifying topological type of singularities using the linear part
Deriving and analyzing a phase portrait
Proving existence or non-existence of periodic orbits
Proofs of elementary results