MATH 7065: Topics in Applied Algebra and Algebraic Structures

The main topics are inner product spaces, orthogonal similarity and geometry (by way of linear algebra). The course is based on Irving Kaplansky’s book “Linear Algebra and Geometry — A Second Course.”

Topics to review

The exam will be based on the first two chapters of Kaplansky’s book.

Inner products in the form of symmetric bilinear forms. For a finite dimensional vector space $V$, know how to prove equivalences for when $(\cdot, \cdot)$ is nonsingular (and when singular). For a two-dimensional vector space $W$ (not characteristic 2) with nonsingular inner product $(\cdot, \cdot)$, know what is meant by $W$ is a hyperbolic plane, how to prove when $W$ under inner product $(\cdot, \cdot)$ is hyperbolic, how to find a basis $\{u, v\}$ where $(u, u) \neq 0$ and $(v, v) \neq 0$ when $W$ is hyperbolic, etc..

Hermitian inner products over vector spaces where the field $F$ has an involution $\ast$: $(y, x) = (x, y)^\ast$, $(ax, y) = a(x, y)$ and $a^\ast(x, y) = (x, ay)$

For finite dimensional vector space $V$ of dimension $n$ and inner product $(\cdot, \cdot)$ (symmetric or Hermitian), know what is meant by $B$ is an orthogonal basis. Also be able to prove that if $(\cdot, \cdot)$ is nonsingular on $V$ and $S$ is an $r$ dimensional subspace where the restriction of $(\cdot, \cdot)$ to $S$ is nonsingular, then $S \cap S^\perp = (0)$ and $V = S \oplus S^\perp$.

Be able to establish basic results about finite dimensional vector spaces over real closed fields, including results about self-adjoint transformations, invariant subspaces, etc. (like can always find an orthonormal basis). Know what can be said (and proved) about $T$, $TT^\ast$, $T^\ast T$ etc. on unitary spaces, what “normal” means.

There will not be questions about quadratic forms in characteristic 2 – so no Arf invariants.