Real Analysis I: MATH 8143
August 2018 Exam

Real Numbers
• The Field, Positivity, and Completeness Axioms (1.1)
• The Natural and Rational Numbers (1.2)
• Countable and Uncountable Sets (1.3)
• Open, Closed, and Borel Sets (1.4)
• Sequences (1.5)
• Continuous Real-Valued Functions (1.6)

Lebesgue Measure
• Lebesgue Outer Measure (2.2)
• Lebesgue Measurable Sets (2.3)
• Outer and Inner Approximation (2.4)
• Countable Additivity, Continuity, and the Borel-Cantelli Lemma (2.5)
• Nonmeasurable Sets (2.6)
• The Cantor Set and the Cantor-Lebesgue Function (2.7)

Lebesgue Measurable Functions
• Sums, Products, and Compositions (3.1)
• Sequential Pointwise Limits and Simple Approximation (3.2)
• Egoroff’s Theorem and Lusin’s Theorem (3.3)

Lebesgue Integration
• The Lebesgue Integral for bounded functions on sets of finite measure (4.2)
• The Lebesgue Integral for nonnegative functions (4.3)
• The General Lebesgue Integral (4.4)
• Countable Additivity and Continuity of Integration (4.5)
• Uniform Integrability and the Vitali Convergence Theorem (4.6)

Further topics in Lebesgue Integration
• Uniform Integrability and Tightness (5.1)
• Convergence in Measure (5.2)

Differentiation and Integration
• Continuity of Monotone Functions (6.1)
• Differentiability of Monotone Functions (6.2)
• Functions of Bounded Variation (6.3)
• Absolutely Continuous Functions (6.4)
• Integrating Derivatives and Differentiating Indefinite Integrals (6.5)