Homework Set 1: Riemann Sums

1. For both part (a) and (b), estimate the area under the graph of \( f(x) = \frac{1}{x} \) from \( x = 1 \) to \( x = 5 \) using four approximating rectangles and the indicated endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   a. Right endpoints.
   b. Left endpoints.

2. For both part (a) and (b), estimate the area under the graph of \( f(x) = 25 - x^2 \) from \( x = 0 \) to \( x = 5 \) using five approximating rectangles and the indicated endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   a. Right endpoints
   b. Left endpoints
3. (a) Use: \[ A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} [f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x] \]

to find an expression for the area under the curve \( y = x^3 \) from 0 to 1 as a limit.

(b) Use: \[ 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]
to evaluate the limit in part (a).

4. If \( f(x) = \ln x - 1, \ 1 \leq x \leq 4 \), evaluate the Riemann Sum with \( n = 6 \), taking the sample points to be the left endpoints. (Give your answer correct to 6 decimal places.) What does the Riemann Sum represent? Illustrate with a diagram.

5. Use the limit definition of the definite integral to evaluate \( \int_{-1}^{2} (2 - x^2) \, dx \) using Right Riemann Sums.
6. Use the limit definition of the definite integral to evaluate the area between the two curves 
\( y = x \) and \( y = x^2 \) using Right Riemann Sums.

For questions 6 and 7, express the limit as its corresponding definite integral on the given interval.

7. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1+x_i} \Delta x, \ [1, 5] \)

8. \( \lim_{n \to \infty} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^3 \frac{1}{n}, \ [1, 2] \)

For questions 8 and 9, express the definite integral as its corresponding Right Riemann Sum.

9. \( \int_{1}^{10} (x - 4 \ln x) \, dx \)

10. \( \int_{2}^{4} (x^6 - 3x^2) \, dx \)