Homework Set 4: Volumes

1. Use the shell method to find the volume of the solid obtained by rotating the region bounded by \( y = 8x - x^2 \) and \( y = 0 \) about the y-axis.

2. Let \( S \) be the solid obtained by rotating the region bounded by \( y = x(x - 1)^2 \) and \( y = 0 \) about the y-axis. Explain why it is awkward to use slicing to find the volume of \( V \) of \( S \). Then find \( V \) using the shell method. Note: It may help to draw a picture of the situation.
II Volume of Solids: To find the volume $V$ of a solid, perform one of the following 2 methods:

- Vertical slicing, find the area $A$ of the slice as a function of $x$, and integrate with respect to $x$:
  \[ V = \int_{a}^{b} A(x) \, dx \]

- Horizontal slicing, find the area $A$ of the slice as a function of $y$, and integrate with respect to $y$:
  \[ V = \int_{a}^{b} A(y) \, dy \]

3) Consider the region $\mathcal{R}$ bounded by $y = 9 - x^2$, $y = 0$, and $x = 0$ (in the first quadrant). (Note: you may use MATH 9 and your answers should be correct to 4 decimal places)

a) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the $x$-axis.

Vertical or Horizontal slicing? 
Slice is a Disk or a Washer? 
\[ V = \int_{a}^{b} A(x) \, dx \quad \text{or} \quad V = \int_{a}^{b} A(y) \, dy \]

Area of a slice $A =$

Volume $V =$

b) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = 9$.

Vertical or Horizontal slicing? 
Slice is a Disk or a Washer? 
\[ V = \int_{a}^{b} A(x) \, dx \quad \text{or} \quad V = \int_{a}^{b} A(y) \, dy \]

Area of a slice $A =$

Volume $V =$
c) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = -1$.

Vertical or Horizontal slicing?
Slice is a Disk or a Washer?

$$V = \int_a^b A(x) \, dx \quad \text{or} \quad V = \int_a^b A(y) \, dy$$

Area of a slice $A =$

Volume $V =$

d) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the $y$–axis.

Vertical or Horizontal slicing?
Slice is a Disk or a Washer?

$$V = \int_a^b A(x) \, dx \quad \text{or} \quad V = \int_a^b A(y) \, dy$$

Area of a slice $A =$

Volume $V =$

e) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $x = 3$.

Vertical or Horizontal slicing?
Slice is a Disk or a Washer?

$$V = \int_a^b A(x) \, dx \quad \text{or} \quad V = \int_a^b A(y) \, dy$$

Area of a slice $A =$

Volume $V =$
f) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $x = 4$.

Vertical or Horizontal slicing?
Slice is a Disk or a Washer?
$V = \int_{a}^{b} A(x) \, dx$ or $V = \int_{a}^{b} A(y) \, dy$ ?
Area of a slice $A = $

Volume $V = $

4) Consider the region $\mathcal{R}$ bounded by the curves $y = x^2$ and $y = x$. Sketch the graph and set up a formula to find the volume of the solid obtained by rotating $\mathcal{R}$ about the following axes:

a) About the x-axis.

b) About the y-axis.

c) About the line $x = 1$. 
5. The base of a certain solid is the area bounded above by the line \( y = f(x) = 9 \) and below by the curve \( y = g(x) = 16x^2 \). Cross-sections perpendicular to the x-axis are squares. (see figure to right)

Use the formula \( V = \int_a^b A(x) \, dx \) to find the volume of the solid.

The side of the square cross-section is a function of x, given by \( s(x) = \). Thus, the volume of the solid is \( V = \).

6. The base of the solid \( S \) is an elliptical region which is bounded by the curve \( 9x^2 + 4y^2 = 36 \). Cross-sections perpendicular to the x-axis are isosceles right triangles whose hypotenuse is on the base of the solid. Find the volume of \( S \). You may find drawing a diagram helpful.