Homework Set 2: Arc Length & the Average Value of a Function

Sketch a graph of the given curve and find its exact length using the Arc Length formula.

1. \( x = 1 + t^3, \ y = 3 - 2t^2, \ 0 \leq t \leq 2. \)

2. \( x = e^t + t, \ y = 4e^{t/2}, \ -2 \leq t \leq 1. \)

3. \( x = 4 \cos t, \ y = 6 \sin t, \ 0 \leq t \leq 2\pi. \) (Hint: use MATH 9 to calculate the integral.)

4. \( x = y^{5/2}, \ 0 \leq y \leq 1. \)
5. Consider:  \( x = r \cos t, \ y = r \sin t, \ 0 \leq t \leq 2\pi, \) where \( r \) is any positive real number.

a. Sketch the graph of the curve.

The graph is a ______________________

b. Find the arc length of this figure. (Leave your answer in terms of \( \pi \).)

\( x' = \underline{\underline{}} \quad \text{and} \quad y' = \underline{\underline{}} \)

The length is \( L = \int_{0}^{2\pi} \sqrt{\left( \frac{\underline{\underline{}}}{} \right)^2 + \left( \frac{\underline{\underline{}}}{} \right)^2} \, dt \)

\( = r \int_{0}^{2\pi} \sqrt{\left( \frac{\underline{\underline{}}}{} \right)^2 + \left( \frac{\underline{\underline{}}}{} \right)^2} \, dt \quad \text{by factoring } r \text{ out} \)

\( = \)

c. Is this the expected answer?

6. Find the average value of \( g(x) = x^2\sqrt{1 + x^3} \) on the interval \([0, 2]\).

7. Find the number(s) \( b \) such that the average value of \( f(x) = 5 - 6x + 3x^2 \) on the interval \([0, b]\) is equal to 3.
8. If \( f(x) \) is continuous and \( \int_1^5 f(x) \, dx = 12 \), show that \( f(x) \) equals 3 at least once on the interval \([1, 5]\).

9. Suppose \( f(x) = (x - 5)^2 \)
   a. Find the average value of \( f(x) \) on the interval \([3, 6]\)
   b. Find \( c \) such that \( f_{AVG} = f(c) \).
   c. Sketch the graph of \( f(x) \) and a rectangle whose area is the same as the area under the graph of \( f(x) \).

10. If a cup of coffee has temperature 95°C in a room where the temperature is 20°C, then, according to Newton’s Law of Cooling, the temperature of the coffee after \( t \) minutes is \( T(t) = 20 + 75e^{-t/50} \). What is the average temperature of the coffee during the first half hour?