Homework Set 4: Definite Integrals

Use the properties of integrals to solve.

1. Given that \( \int_4^9 \sqrt{x} \, dx = \frac{52}{13} \), then \( \int_4^9 \sqrt{x} \, dx = \)

2. \( \int_{2\pi} \, x^3 \cot x \, dx = \)

3. Write as a single integral in the form \( \int_a^b f(x) \, dx \):
   \[
   \int_{-5}^{1} f(x) \, dx + \int_{1}^{3} f(x) \, dx - \int_{-1}^{3} f(x) \, dx =
   \]

4. If \( \int_2^7 f(x) \, dx = 8 \) and \( \int_2^5 f(x) \, dx = 3.9 \), then \( \int_5^7 f(x) \, dx = \)

5. If \( \int_0^8 f(x) \, dx = 23 \) and \( \int_0^8 g(x) \, dx = 8 \), then \( \int_0^8 [2f(x) + 3g(x)] \, dx = \)

6. Use the graph of \( f(x) \) shown to evaluate the integrals.
   a. \( \int_0^3 f(x) \, dx = \)
   b. \( \int_3^7 f(x) \, dx = \)
   c. \( \int_0^8 f(x) \, dx = \)
Evaluate the integral by interpreting it in terms of area. Draw a graph of the situation (if not given).

7. \( \int_0^5 \sqrt{25 - x^2} \, dx \)

8. \( \int_{-1}^2 (2x - 1) \, dx \)

9. \( \int_{-1}^3 |x| \, dx \)

Evaluate the integral by using antiderivatives.

10. \( \int_{-1}^2 x^5 \, dx \)

11. \( \int_0^2 2t^3 - 4t - 1 \, dt \)
12. \( \int_0^4 (3u - 1)(2u + 5) \, du \)

13. \( \int_1^2 \frac{y + 4y^6}{y^3} \, dy \)

14. \( \int_0^1 x^2 (\sqrt[3]{x} - \sqrt[5]{x}) \, dx \)

15. The area of the region that lies to the right of the y-axis and to the left of the parabola \( x = 4y - 2y^2 \) (the shaded region in the figure) is given by the integral \( \int_0^2 (4y - 2y^2) \, dy \). (Turn your head clockwise and think of the region as lying below the curve \( x = 4y - 2y^2 \) from \( y=0 \) to \( y=2 \).) Find the area of this region.