Homework Set 12
(Sections 7.4 & 7.5)

For questions 1 and 2, find the convolution $f(t) \ast g(t)$.

1. $f(t) = t$, $g(t) = 1$

2. $f(t) = \sin t$, $g(t) = \sin t$

For questions 3 and 4, use the convolution thereom: $\mathcal{L}\{f(t) \ast g(t)\} = \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\}$ to find the inverse Laplace transforms of the given functions.

3. $F(s) = \frac{1}{s(s^2+4)}$

4. $F(s) = \frac{1}{(s^2+1)^2}$
For questions 5 through 8, use either $\mathcal{L}\{tf(t)\} = -F'(s)$ (and related $\mathcal{L}^{-1}\{F(t)\} = -\frac{1}{s} \mathcal{L}^{-1}\{F'(t)\}$) or $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_{s}^{\infty} F(\sigma) \, d\sigma$ (and related $\mathcal{L}^{-1}\{F(t)\} = t \mathcal{L}^{-1}\{\int_{s}^{\infty} F(\sigma) \, d\sigma\}$) to find the Laplace transforms or the inverse Laplace transforms of the given functions as indicated. [note: don't forget to check the extra condition needed in order to use the second rule.]

5. $f(t) = te^{-t} \sin t$

6. $f(t) = \frac{e^{3t} - 1}{t}$

7. $F(s) = \ln\left(\frac{s-2}{s+2}\right)$
8. \( F(s) = \tan^{-1}\left(\frac{3}{s+2}\right) \)

9. Solve: \( tx'' + (t - 2)x' + x = 0, \ x(0) = 0 \) [hint: remember to use \( \mathcal{L}\{tf(t)\} = -F'(s) \) ]

For questions 10 and 11, find the inverse Laplace transform of the given functions and then sketch a graph of \( f(t) \)

10. \( F(s) = \frac{e^{-s}}{s+2} \)

11. \( F(s) = \frac{1-e^{-2\pi s}}{s^2+9} \)
12. Find $\mathcal{L}\{g(t)\}$ where the function $g(t)$ is a piecewise function defined by

$$g(t) = \begin{cases} 
0 & \text{if } 0 \leq t < 5 \\
t^3 + 1 & \text{if } t \geq 5
\end{cases}$$

13. (Extra Credit) Find $\mathcal{L}\{g(t)\}$ where the function $g(t)$ is a piecewise function defined by

$$g(t) = \begin{cases} 
\cos\left(\frac{1}{2} \pi t\right) & \text{if } 3 \leq t \leq 5 \\
0 & \text{if } t > 5 \text{ or } t < 3
\end{cases}$$